ParCFD'2020
32^{nd} International Conference on Parallel Computational Fluid Dynamics
May-11-13 2020, Nice FRANCE

A PARALLEL SEMI-COUPLED PROJECTION MODEL FOR A LONG-TERM MORPHODYNAMIC EVOLUTION ON UNSTRUCTURED GRIDS

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Key words: Sediment transport, shallow-water equations, block-domain decomposition, MPI, unstructured finite-volume method

Abstract. In this work, we present a numerical method for long-time sediment evolution governed by the two-dimensional shallow water equations semi-coupled with the Exner sediment balance equation. The method is based on Chorin's projection, which combines the momentum and continuity equations in order to establish a Poisson-type equation for the water surface level including the bed evolution. The equations are discretized using an unstructured finite-volume technique. In order to obtain fast simulations on long-term evolving morphodynamic problems, a parallelization strategy is also presented using a block domain decomposition technique and Message Passing Interface. A comparison with analytical and existing numerical results are done to test the performance of the proposed method.

1 INTRODUCTION

Many numerical formulations have been proposed to model sediment transport on shallow-water flows. The suitable numerical strategy depends on the physical and mathematical nature of the problem [1]. In general, coupled strategies has shown to be more stable than uncoupled methods. However, it has the inconvenience of implementation complexities, high numerical cost and excessive diffusion in the evolution of the bed. On the other hand, semi-coupled methods have many of the advantages of uncoupled methods with more stability, a better response to high dependence fluid-sediment and also to super-critical fluids [2].

Although many works have been done using rectangular grids, the use of unstructured meshes is necessary for real applications. This work presents a parallel semi-coupled approach for solving the Saint-Venant-Exner system based on a projection method (PM) and a second-order unstructured finite-volume method (UFVM) [3]. It splits up the velocity field and water surface level resulting in a Poisson-type problem for the water surface level which includes the bed evolution in its equation. If the fluid-sediment interaction is weak, then the method becomes decoupled and the simulations can proceed for a long time without numerical dissipation.

Numerical results on sediment transport using slow varying flows are obtained on days, months or even years of real time simulation. Thus simulations using a UFVM requires huge computing power [4]. Motivated by this, we present a parallel algorithm for the proposed sediment transport model. It is formulated by using a block domain decomposition and interprocessor data communication techniques with Message Passing Interface. The unstructured-grid cells are reordered according to the neighboring relations and decomposed into blocks using a load-balanced distribution to give all processors an equal amount of elements. Moreover, resulting linear systems are solved using a parallel Multi-Color Successive Over-Relaxation (MSOR) method.

2 NUMERICAL RESULTS

The proposal parallel UFVM is initially tested using well-known benchmarks such as the sediment evolution of a conical dune of sand in a rectangular channel. For this case, the sediment bed solution evolves towards a smooth star-shaped pattern expanding over time with a given spreading angle. In this example, the Exner equation with the Grass formula is considered with coefficient $A_g = 0.001$ s^2/m. Numerical solutions are shown in Fig. 1 at $t = 360,000$ s using an unstructured mesh of around 12,000 cells. The proposed method produces an angle of spread close to the theoretical one (21.7867893^o) showing the correct implementation of the model.

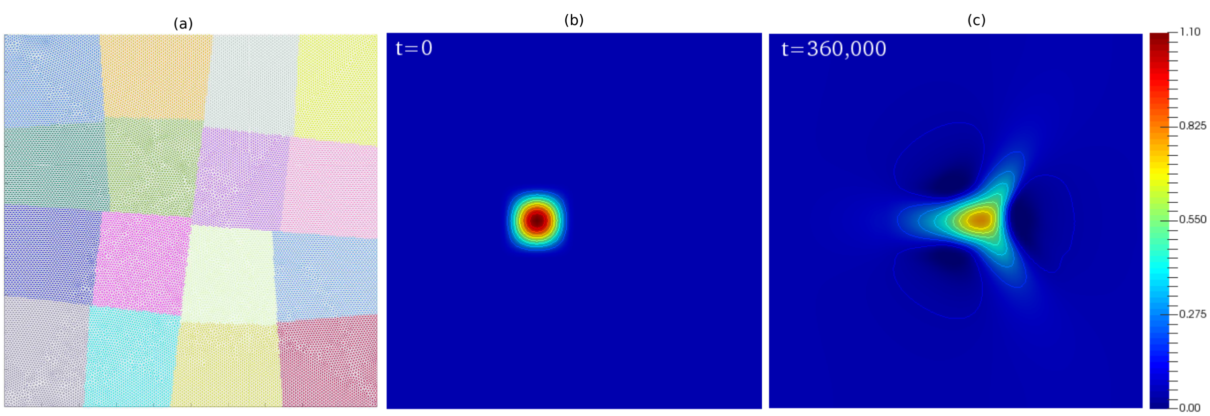

Figure 1: Initial and numerical solution of the bed level at $360,000$ s for the sediment evolution of a conical dune of sand using an unstructured mesh with a block-domain decomposition.

In the conical dune example, $\Delta t = 0.5$ is selected, thus 720,000 time steps are required to obtain the final results. It is important to remark that time steps can be large due to the implicit formulation; however, a linear system has to be solved at each stage corresponding to the water-level Poison equation. It makes simulations slow, particularly for fine meshes. For example, 50,000 seconds of a sequential simulation for 40,690 cells and $\Delta t = 0.05$ can take almost 17 hours of CPU time.

To further demonstrate the capabilities of the proposed model, the numerical method has been set up for predicting the morphodynamics in a coastal area located at the northern coast of Yucatan, Mexico. The study site includes a 6-km-long Progreso pier, as shown in Fig. 2(a). Although, erosion along this area is attributed to storm passage, the harbor development have significantly contributed to erosion. Motivated by this, long-term sediment transport studies are important to understand the influence of this structure into the erosion problem. Moreover, it is expected that numerical simulations with the actual flow topology will test the limits of the proposed method and provide useful feedback for further improvements. The computational domain is an area of around 10 km $\times$ 10 km. Next, results are shown using an unstructured mesh of $25,368$ cells and using $\Delta t = 0.5$, as shown in Fig. 2(b).

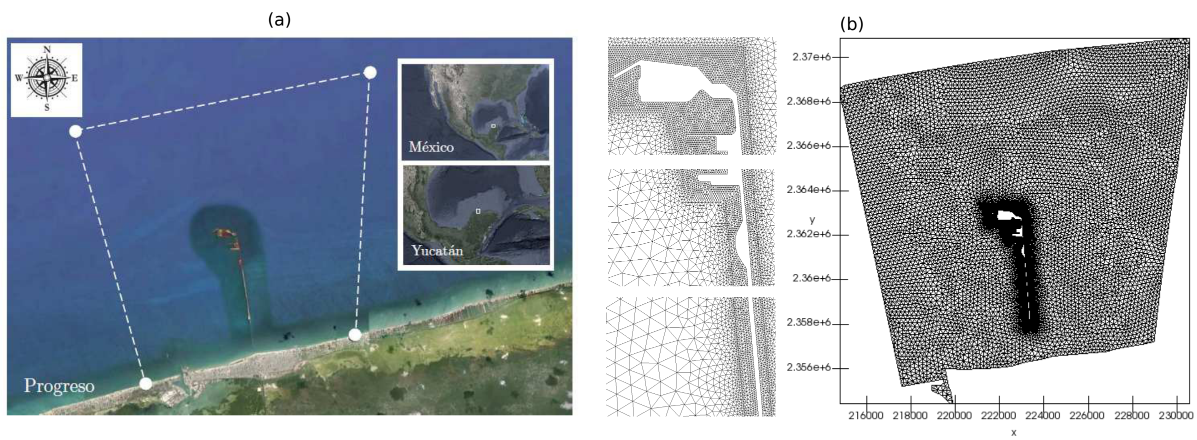

Figure 2: (a) The 6-km-long Progreso pier located at the northern coast of Yucatan, Mexico, and (b) unstructured mesh of 25,368 cells used in this example.

The current shallow-water model can be applied because the depth of the study area does not extent the 13 m depth (see Fig. 3(a)). The sea bottom sediments are categorized as fine grained sand. For further details regarding the local topology and climatology, the reader is directed to [5]. The flow is directed from left to right and the beach is located on the bottom boundary. Initial results are shown in Fig. 2(b) after 12 hours of simulation. The results of the velocity field agree quite well with the expected results. An steady state velocity field is reached after few hours depending of the inflow velocity. However, the bathymetry distribution presents only small changes after some days of simulation. Current work is devoted to use the new parallel algorithm to analyze the sediment evolution after few months (or years) for this example.

3 CONCLUSIONS

In this work, we present an UFVM for solving a semi-coupled model of bed load transport in shallow-water flows. In order to obtain a faster simulations, a parallel code

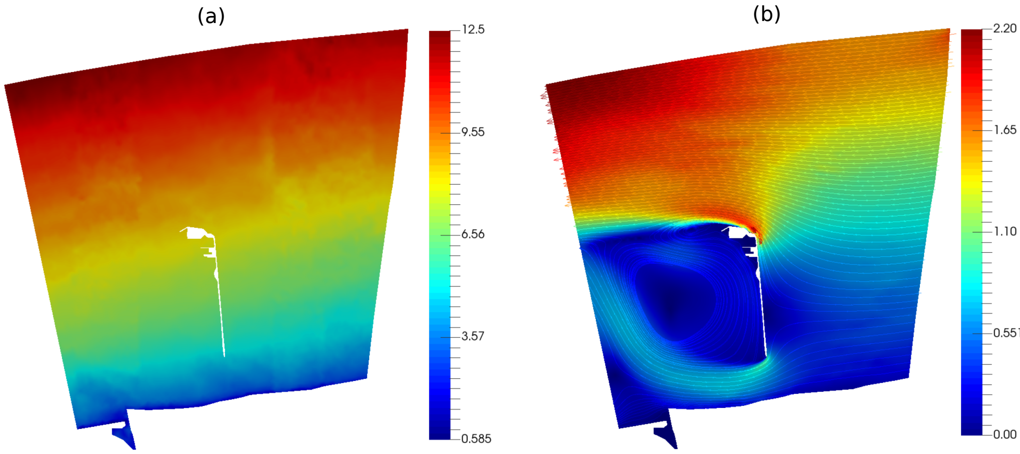

Figure 3: (a) Bathymetric data used in the numerical model. (b) Numerical velocity field after 12 hours of simulation.

is proposed based on block-domain decomposition and a parallel MSOR linear system solver. Verification of the method is carried out using well-known benchmarks for sediment transport. The numerical results exhibited high accuracy, stability, and good performance behavior for the hydraulic regimes considered. To demonstrate the capabilities of the proposed model, it is applied for long-term simulations of flows and morphological evolution in a coastal zone. Initial results are presented for this example; however, computational and performance results will be further analyzed in a future work.

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