

CORRECTION OF THE OCEAN DYNAMICS CALCULATIONS USING THE NEMO MODEL BY MEANS OF ASSMILATION OF ALTIMETRY DATA FOR THE ATLANTIC

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Summary. Using the earlier proposed data assimilation method called Generalized Kalman Filter (GKF) and the model of ocean circulation Nucleus for European Modelling of the Ocean (NEMO), the main ocean parameters in the Atlantics are calculated. The state and variability of the ocean level, surface and subsurface water temperature, zonal and meridional components of current velocity are analyzed. The results of calculations after data assimilation are compared with the analogous calculations using the initial model without data assimilation. Separately, the characteristics of currents after data assimilation are estimated and it is shown that the calculated quantities become more dynamic and have a higher variability.

1 INTRODUCTION

The data assimilation (DA) theory is a rapidly developing cross-disciplinary domain involving mathematics, informatics, and geosciences (physics of atmosphere, oceanology, and climatology). The main objective of this theory and practical methods created and developed on its basis is to correct the results of the model calculation with consideration of both the physics of the processes and the observed information with the aim of providing the best forecast of the state and variability of simulated physical processes. This task is not simple, if we take into account the complicity of the models of dynamics themselves, as well as the inhomogeneity and insufficiency of the initial observational data. Therefore, the description of both the DA methods themselves and their practical implementation take a significant

place in the scientific literature [1-4].

In this work, we use a recently developed authors' data assimilation method called Generalized Kalman filter (GKF), which is described in detail in [5,6]. Its principal advantages in comparison with analogous DA methods are its capability to consider not only the closeness of the observations and the model and to perform data assimilation on this basis, but also to consider the dynamics of the model itself and the temporal variability of observed values. It is shown in [6] that in comparison with the commonly known Ensemble Optimum Interpolation (EnOI) method, the GKF method provides a considerable quantitative improvement in the variance of the forecast estimate and in the variance of the analysis error.

The main model of ocean dynamics used in this work is the NEMO model developed in the Pierre Simon Laplace Institute (Paris) and described in many publications (see, for example, [7]). The NEMO model is well known [4]; it is often used to calculate the ocean dynamics in different regions of the World Ocean. For example, we can note paper [8], in which the NEMO model was applied to study the ice dynamics in the Russian sector of the Arctic.

In this work, the NEMO model was used jointly with the GKF method to study the dynamics of the Central and North Atlantic. As the archive of observed information, the known AVISO (Archiving, Validating and Interpolating Satellite Ocean data) archive was used; the observational data are available on the website www.aviso.org.

2 BRIEF MATHEMATICAL DESCRIPTION OF DA METHOD GKF

Let the mathematical model be governed by the system of equations:

$$\frac{\partial X}{\partial t} = \Lambda(X, t) \quad (1)$$

with the initial condition $X(0) = X_0$, where X is the model state vector defined on a phase space, i.e. on the set of values which the model variables can take, Λ denotes the vector-function defined on the same phase space and on a time interval $[0, T]$. In the discrete realization, the model state vector has a dimension r , where r is the number of grid points multiplied by the number of independent model variables. On the time interval $[0, T]$, the discretization $0 = t_0 < t_1 < \dots < t_N = T$ is introduced. For simplicity and also without loss of generality, all these moments are assumed to be equidistant, $\Delta t = t_{n+1} - t_n$. On each time interval $[t_n, t_{n+1}]$, $n = 0, 1, \dots, N - 1$, the model equations are numerically solved and at the moment t_{n+1} , data assimilation is performed by using formulae [8]

$$X_{a,n+1} = X_{b,n+1} + K_{n+1}(Y_{n+1} - HX_{b,n+1}), \quad (2)$$

$$K_{n+1} = (\sigma_{n+1}^2)^{-1}(\Lambda_{n+1} - C_{n+1})(H\Lambda_{n+1})^T Q_{n+1}^{-1}, \quad (3)$$

$$\sigma_{n+1}^2 = (H\Lambda_{n+1})^T Q_{n+1}^{-1} (H\Lambda_{n+1}), \quad (4)$$

where $X_{a,n}, X_{b,n}$, $n = 0, 1, \dots, N$ are the model state vectors before and after assimilation, respectively, i.e., the analysis and background; it is assumed that $X_{a,0} = X_{b,0} = X_0$ is the known initial condition; Y_n is the observation vector at the same instant of time that has a

dimension m , where m is the number of observation points multiplied by the number of independently observed variables; K_n is the gain matrix (analogous to the Kalman gain matrix) with a dimension $r \times m$; H is the observational projection matrix with a dimension $m \times r$. Parameters C_{n+1} and Q_{n+1}^{-1} are determined from the ensemble of model calculations, as it was made in [5,6].

3 COMPUTATIONAL EXPERIMENTS

The authors have performed the installation and adaptation of the NEMO software package on the K-60 high-performance computer (HPC) in the Keldysh Institute of Applied Mathematics of the Russian Academy of Sciences (Moscow, Russia).

In this work, the preliminary calculations with the NEMO model were performed for January 2010. These data were used to create the ensemble and to determine parameters C_{n+1} and Q_{n+1}^{-1} . After that, the fields of model characteristics were corrected by using formulae (2)-(4).

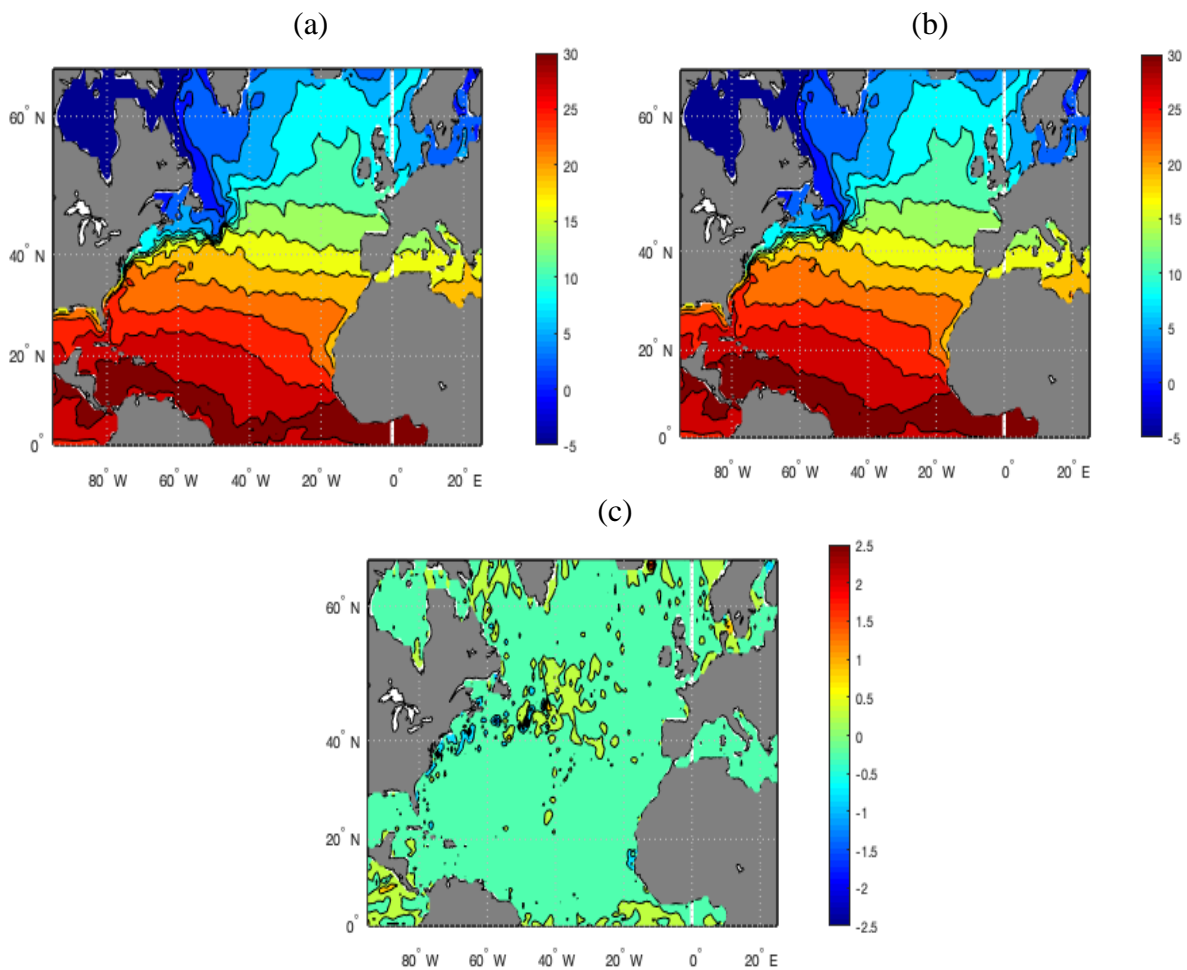


Figure 1: The sea surface temperature (SST) fields ($^{\circ}\text{C}$) calculated by using the NEMO model: a) with DA by the GKF method; b) with no DA (the control calculation); c) field difference (a) – (b) on January 10, 2010.

Figure 1 shows the sea surface temperature (SST) fields calculated using the NEMO model with DA by the GKF method (Figure 1a), without DA (control calculation) (Figure 1b) and their difference (Figure 1c) on January 10, 2010.

It is seen that the fields are similar in general; both calculations reflect the principal large-scale structure of the thermal fields in the Central and North Atlantic. The main difference between the calculations of temperature with a maximum of about 0.5°C is concentrated in the central region of the North Atlantic and in the coastal regions. This difference can be explained by the fact that in the central region, where the vortices related to the Gulf Stream and its southern branch are clearly observed, there is a rise of the ocean level, which is well seen from satellites, however, relatively poorly simulated by the model. Therefore, there are the differences in the SST fields. In the coastal zones, there are numerous wave processes associated with the coastal-trapped waves and their propagation along the shelf, which is poorly reconstructed by the large-scale model.

4 CONCLUSIONS

In this work, the calculations using the known NEMO model applied to the new DA method GKF are performed. The results of calculations show that the GKF method estimates reliably the incoming information and corrects appropriately the model calculations. The model fields of characteristics obtained before and after the correction agree with both the observations and the known tendencies and phenomena occurring in the nature.

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