ParCFD'2020
32nd International Conference on Parallel Computational Fluid Dynamics
May 11-13 2020, Nice FRANCE

OCEAN ACOUSTIC PROPAGATION NUMERICAL SIMULATION BASED ON CHEBYSHEV SPECTRAL METHOD

YONGXIAN WANG*, WEI LIU, WENBIN XIAO, QIANG LAN, XINGHUA CHENG

National University of Defense Technology
College of Meteorology and Oceanography
410073 Changsha CHINA
e-mail: yxwang@nudt.edu.cn, liuweinudt@126.com, hgxiaowb727@126.com, lanqiang_nudt@163.com, chengxinghua@nudt.edu.cn

Key words: Spectral method, Chebyshev polynomial, Ocean sound propagation, High performance computing.

Summary. In this extended abstract, a Chebyshev spectral method with global approximation is used to solve the problem of ocean sound propagation problem. The numerical experiment is tested by using normal modes model of Munk ocean waveguide problem and the results are compared with that of the finite difference method. It shows that the proposed method has the advantages of fast convergence, high accuracy and low computational overhead.

1 INTRODUCTION

In the past decades, finite difference method (FDM) was widely used in developing the normal mode method for the simulation of an underwater acoustic field [1-3]. However it still has many limits. The FDM is inflexible when handling boundary value problems with complex shapes and constructing a high-accuracy scheme. When using the FDM in the underwater acoustics applications, situation several discreting grid points should be arranged in each wavelength which size is in meter-scale typically, thus resulting in huge size of grid points as well as a huge amount of computation and memory's cost for those far-distance ocean applications. The spectral method is another classical, high order and widely used technique to solve partial differential equations. Among them, the Chebyshev spectral method (CSM) has been widely used in meteorology, physics, and engineering fields [4-6]. The greatest advantage of the CSM is that its error can converges exponentially.

In this extended abstract, we attempts to introduce the CSM to solve underwater sound propagation problems.

2 NORMAL MODE MODEL OF UNDERWATER ACOUSTIC PROPAGATION

Consider a homogeneous Helmholtz equation for a 2D horizontally layered medium with depth z and horizontal distance r:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right)+\rho(z)\frac{\partial}{\partial z}\left(\frac{1}{\rho(z)}\frac{\partial p}{\partial z}\right)+\frac{\omega^2}{c(z)^2}p=0 \qquad (1)$$

where $p(r,z)$ is the sound pressure, $\rho(z)$ and $c(z)$ are the density and sound velocity of the seawater, respectively, and ω is the angular frequency of the sound source. Using the technique of separation of variables, we seek a solution of Eq.(1) in the form $p(r,z)=u(z)$ $v(r)$. Then the following modal equation formed:

$$\rho(z)\frac{\partial}{\partial z}\left(\frac{1}{\rho(z)}\frac{\partial u(z)}{\partial z}\right)+\left(\frac{\omega^2}{c(z)^2}-k_r^2\right)u(z)=0 \qquad (2)$$

k_r^2 is an (unknown) constant. Eq. (2) is known as a model equation which forms an eigenvalue problem. In combination with proper boundary conditions at the sea surface $z = 0$ and sea floor $z = H$, the model equation has a series of normal mode solutions $\left(k_{rm}^2, u_m(z)\right), m = 1,2, \ldots$, These normal modes are orthogonal to each other and can form a complete set of standard orthogonal bases. The final sound pressure solution can thus be obtained:

$$p(r,z) = \frac{i}{4\rho\left(z_s\right)} \sum_{m=1}^{\infty} u_m(z_s)\, u_m(z) H_0^{(1)}\left(k_{rm} r\right)$$

where z_s is the sound source depth, and $H_0^{(1)}(\cdot)$ is the Hankel function of the first kind.

We use the transmission loss $TL = -20\log_{10}\left(|p(r,z)|/|p_0|\right)$ instead of the sound pressure itself in practical applications, where p_0 is the reference sound pressure at 1 m from the sound source.

3 SOLVING UNDERWATER SOUND PROPAGATION PROBLEM USING THE CHEBYSHEV SPECTRAL METHOD

In the Chebyshev spectral methods (CSM), we approximate the unknown functions using a truncated series of Chebyshev polynomials $T_k(x)$. That is, we use the partial sum of a infinite series $u_N(x)$ to approximate the smooth function u(x) defined on interval [-1, 1]:

$$u(x) \approx u_N(x) = \sum_{k=0}^{N} \widehat{u}_{\mathrm{k}} T_k(x) \tag{3}$$

Where the values $\widehat{u}_{\mathrm{k}}$ are the unknowns, and $u_N(x)$ is the truncated series with N terms remaining. The specific way to determine these unknowns characterizes the spectral method.

The Chebyshev polynomials $T_k(x)$ are also known as test functions, and they form a set of orthogonal bases of a continuous function space. They are given by $T_k(x) = \cos\left(k\cos^{-1}(x)\right)$, and the expansion coefficient of Eq.(3) can be expressed as

$$\widehat{u}_k = \frac{2}{\pi c_k}\int_{-1}^{1} \frac{u(x)T_k(x)}{\sqrt{1-x^2}}dx, \quad c_k = \begin{cases} 2, k = 0 \\ 1, k > 0 \end{cases} \tag{4}$$

Eqs.(4) and (3) are called (forward) Chebyshev transformation and inverse Chebyshev transformation, respectively. These transformations are use to relate the function $u(x)$ in the physical space and the coefficients $\widehat{u}_{\mathrm{k}}$s in the spectral space. In the CSM, the integral and derivatives can also be expressed as its corresponding part in the spectral space. We presents the details in the full version of this extended abstract.

We then consider the finite truncation of the N term expansion of $u(x)$ in Eq.(3), and turn the PDE problem about the unknown function $u(x)$ into a new algebraic problem with $N + 1$ unknowns $\{\widehat{u}_k, k = 0,1,\cdots,N\}$ by applying Chebyshev forward transformation. According to the ways of constructing $N + 1$ equations and dealing the boundary conditions, the CSM can be further classified into Galerkin, Tau, collocation and pseudospectral methods. Here, both Tau and collocation methods are considered. With the help of Eqs. (3) and (4), a linear algebra eigenvalue system (6) is finally formed and can be solved by any mathematical library and solution tool, such as BLAS/LAPACK, MKL, etc.

$$\sum_{l=0}^{N} L_{kl}\widehat{u}_l = k_r^2 \widehat{u}_k, k = 0,1,\ldots,N \tag{6}$$

4 NUMERICAL TESTS AND RESULTS

Two cases configurations (see table 1) are used to evaluate the accuracy and performance of our method by comparing the results of CSM and that of FDM. An ideal fluid waveguide of shallow water environment is used for case A, and it has the analytical solution which can be used for the accuracy comparisons. Case B is a deep sea environment with more complex Munk sound volocity profile used.

	Case A: Shallow water	Case B: (Deep water)
the depth of seawater H	100 m	5000 m
the maximum of range rmax	3 km	100 km
the depth of sound source z_s	36 m	1000 m
the sound source frequency f	20 Hz	50 Hz
Boundary conditions	$p(z=0)=p(z=H)=0$	$p(z=0)=p(z=H)=0$
the density of the water $\rho(z)$	1 g / cm^3	1 g / cm^3
the sound velocity c	1500 m/s	Munk profile (Fig. 1)

Table 1 : The configurations of two cases for numerical simulations

Figure 2 shows the transmission loss (TL) results of analytical solution, FDM and our CSMs. The number of grid points M = 51 for the FDM, and the item number of truncation N = 10 for two CSM methods. Figure 3 shows the error curves of FDM and two CSM methods compared to the analytical solution. It indicates that CSM method has the smaller error than the FDM, and the CSM-Tau method has the highest accuracy among all these three methods. From the error curves shown in figure 4, we can found that both CSMs have faster convergence speed than the FDM, and among them CSM-Tau has the best convergence performance. The transmission loss results of FDM and our CSMs for case B are shown in figure 5. The number of grid points M = 1001 for the FDM, and the item number of truncation N = 200 for two CSM methods.

From the above results, we can infer that the CSM is an effective and high-accuracy method for solving underwater acoustic propagation problems. In fact it is a global approximation method compared to the widely used FDM, and it need much less discreted grid points than the FDM to reach the same level of error tolerance, thus reducing the amount of computations and storage dramatically.

5 CONCLUSIONS

In this extended abstract, the Chebyshev spectral method is applied to solve the normal mode model of underwater acoustic propagation problem. The results of two CSMs and traditional FDM are compared for the cases of shallow water and deep sea environments. It shows that the CSM has the advantages of fast convergence, high accuracy, and low computational overhead. Our next work will include extending the spectral method to other underwater acoustic models and more realizable applications. Furthermore, how to speed the spectral method in the popular high performance computing platform with many-core architecture would be also worthwhile.

ACKNOWLEDGMENTS

This work was supported in part by the National Key Research and Development Program of China under Grant No. 2016YFC1401800, in part by the National Natural Science Foundation of China under Grant No. 61702531, 61972406, 51709267.

REFERENCES

[1] Pekeris C L. *Theory of propagation of explosive sound in shallow water*, Mem. Geol. Soc. Am., 1948, 27:1-116.

[2] Etter P C. *Underwater acoustic modeling and simulation*. CRC Press, 2013.

[3] Porter, Michael B. and Reiss, Edward L., *A numerical method for bottom interacting ocean acoustic normal modes*, JASA 77, 1760-1767 (1985).

[4] A. Quarteroni, A. Valli. *Numerical Approximation of Partial Differential Equations*. Springer-Verlag, 1998.

[5] N P Muravskaya. *Spectral methods in physical and chemical measurements*, Journal of Physics, 2019, 1420: 012005.

[6] Sidi Ammi, Moulay Rchid;Torres, Delfim F.M. *Galerkin spectral method for the fractional nonlocal thermistor problem*, 2017,73:1077-1086.

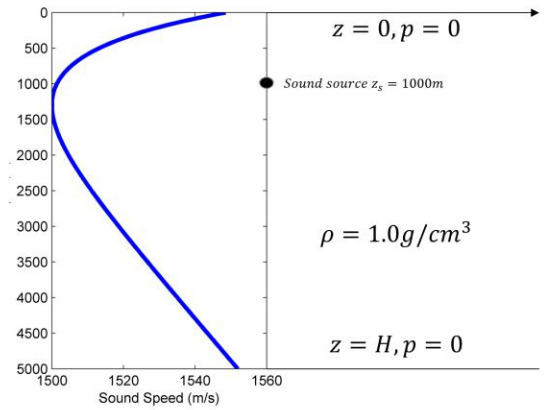
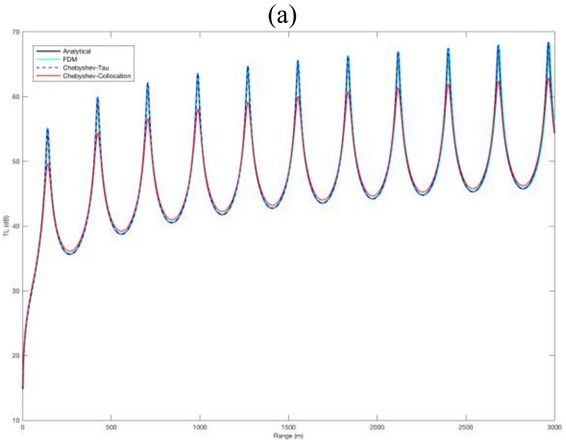


Figure 1: Munk sound velocity profile used in case B

(a)



(b)

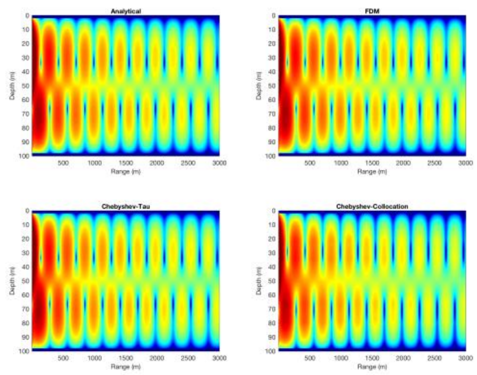


Figure 2: TL results of analytical solution, FDM and our CSM for case A. (a) TLs at depth 36 m; (b) TLs in the whole domain.

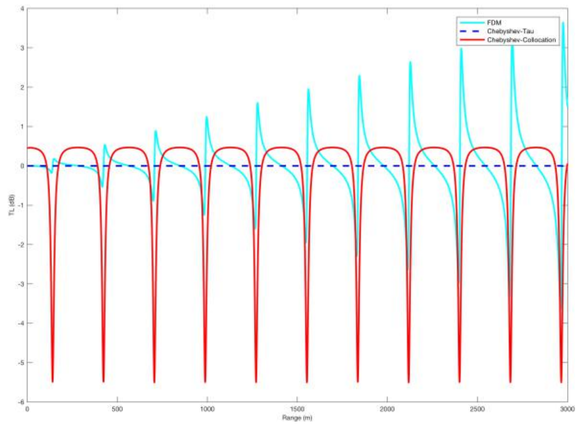


Figure 3: TL error comparison for FDM and two CSM methods.

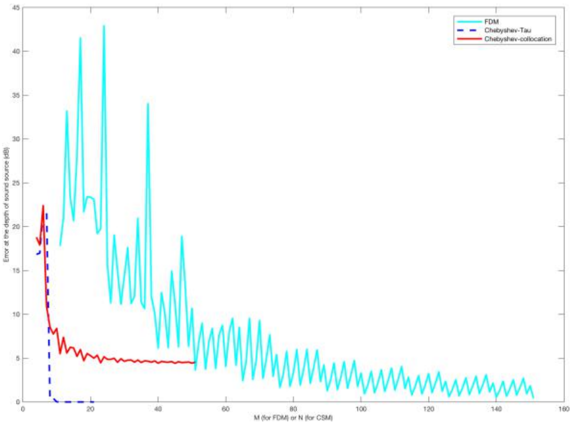
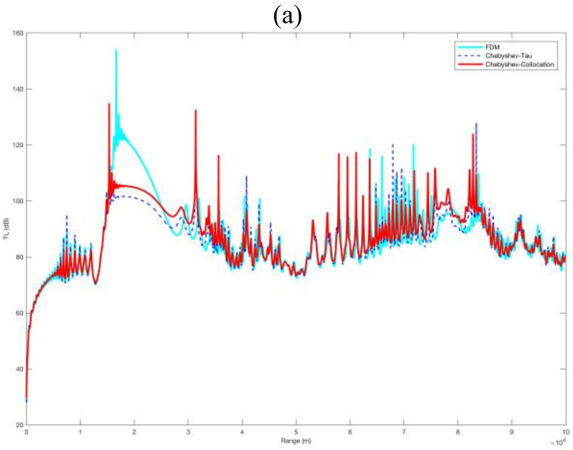


Figure 4: The error curves for case A.

(a)



(b)

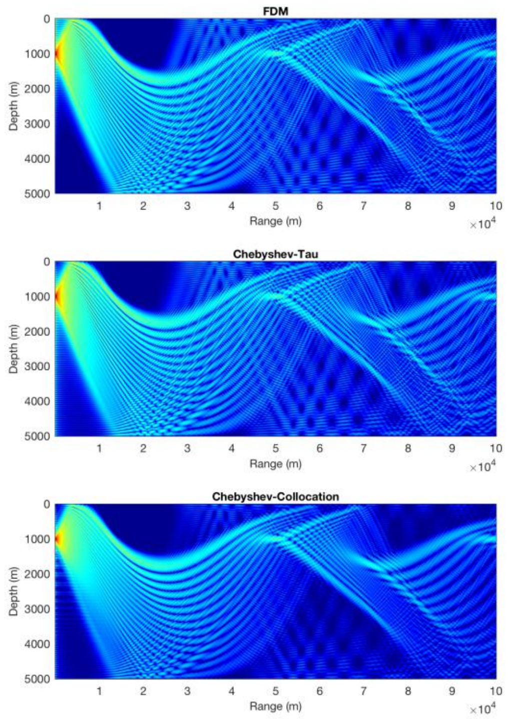


Figure 5: TL results of analytical solution, FDM and our CSM for case B. (a) TLs at depth 1000m; (b) TLs in the whole domain.