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STUDY ON WIND TURBINE POWER UNDER WAKE INTERACTION IN WIND FARM WITH AN ACTUACTOR DISK MODEL

YU-CHENG HSU AND SHIU-WU CHAU

National Taiwan University
Department of Engineering Science and Ocean Engineering
10617 Taipei, TAIWAN
e-mail: chausw@ntu.edu.tw, web page: http://esoe-fsl.strikingly.com

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Summary. This paper employs an actuator disk model to predict the power of wind turbine array in a wind farm. The three-dimensional flow field of a wind farm is governed by the steady continuity and momentum equations along with a two-equation turbulence model where an actuator disk model is used to represent the aerodynamic impact of wind turbine on the incoming airflow via a body force approach. An in-house parallelized code based on a SIMPLE-type algorithm to decouple velocity and pressure appearing in the governing equations is implemented based on a domain decomposition approach. The proposed actuator disk model is first validated with an offshore wind farms to justify the proposed numerical approach in delivering a power prediction with good accuracy. Three different wind conditions are further tested to disclose the influence of wake interaction on the behavior of wind turbine power. Additionally, a simplified wake modeling with single row arrangement is adopted to compare with a complete wind farm simulation to identify their differences.

1 INTRODUCTION

Recently wind energy is growing rapidly in the renewable energy sector where the rotor size dramatically increases to fit the emerging demand of high-power offshore turbines. The energy conversion efficiency is a main issue of wind turbine because of low energy density in airflow along with the high construction cost on the sea. Unfortunately, power loss and emphasized dynamic load of downstream wind turbines in a wind farm is a direct consequence of the wake interaction among wind turbines[1]. For an accurate and efficient prediction of the power behavior of a wind farm, this study proposes an actuator disk method to numerically forecast the power performance of single wind turbine as well as the whole wind farm.

2 ACTUACTOR DISK MODEL

It is assumed that the flow field is incompressible and steady, where gravity and temperature effect are neglected. The continuity and Reynolds-Averaged Navier-Stokes (RANS) equation are given as follows

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0 \tag{1}$$

$$\bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \upsilon \frac{\partial^2 \bar{u}_i}{\partial x_i \partial x_j} - \frac{\partial \overline{u_i' u_j'}}{\partial x_j} + f_i \tag{2}$$

where ρ denotes the density, p the pressure, υ the kinematic viscosity, $\overline{u_i' u_j'}$ the Reynolds stress, f_i the body force representing the aerodynamic characteristic of rotor. A k-ε turbulence model is adopted in the study. The transport equation of turbulent kinetic energy (k) and its dissipation rate (ε) are shown as follows[2]:

$$\frac{\partial}{\partial x_j}(k u_j) = \frac{\partial}{\partial x_j}\left(\left(\upsilon + \frac{\upsilon_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right) + P_k - \varepsilon \tag{3}$$

$$\frac{\partial}{\partial x_j}(\varepsilon u_j) = \frac{\partial}{\partial x_j}\left(\left(\upsilon + \frac{\upsilon_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right) + c_{\varepsilon 1} P_k \frac{\varepsilon}{k} - c_{\varepsilon 2}\frac{\varepsilon^2}{k} \tag{4}$$

$$\upsilon_t = C_\mu \frac{k^2}{\varepsilon} \tag{5}$$

where P_k denotes the production of turbulent kinetic energy defined in Eq.(6), υ_t the turbulence eddy viscosity and $(\sigma_k, \sigma_\varepsilon, c_{\varepsilon 1}, c_{\varepsilon 2})$ the model constants. In the actuator disk model, the wind turbine geometry is fully neglected in the computational domain. Instead, the swept surface of the rotor is replaced with a force distribution exerted by the incoming airflow, where the body force distribution represents the aerodynamic characteristics of the rotor[3]. A Cartesian mesh coupled with a body force distribution alongside a turbulence source term representing the turbulence generated by rotor motion is applied to the analysis of the wake flow behind the wind turbine. With a given (characteristic) inflow velocity, the corresponding rotor speed and power are obtained from the power curve. It is assumed that the blade force is composed of axial force F_a and tangential force F_t that are uniformly distributed over the rotor disk. For a power prediction, P_i and ω_i represents the initial conditions of turbine and U_{in} denotes the inflow velocity. The axial force density f_a is calculated from Eq.(6) and the tangential force density f_t is calculated from Eq.(7):

$$f_a = \frac{P}{\pi R^2 U_{AVE}} \tag{6}$$

$$f_t = \frac{3P}{4\pi^2 R^3 \omega} \tag{7}$$

The body force is first distributed over the rotor disk based on prescribed blade elements with the abovementioned force density, and the force on blade elements is further distributed to the Cartesian cells. In the proposed actuator disk model, a uniform distribution of blade elements on the rotor disk is adopted[4]. The body force is only effective within a region with size of ϵ. Only the Cartesian cell with a distance d to the target blade element smaller than ϵ is considered for body force distribution. The body force is distributed from a blade element to Cartesian cells via a Gaussian distribution $\eta_\epsilon(d)$, Eq.(8).

$$\eta_\epsilon(d) = \frac{1}{\epsilon^2 \pi^{2/3}} \exp\left[-\left(\frac{d}{\epsilon}\right)^2\right] \tag{8}$$

In this study, an in-house Fortran code, WIFA3D, is adopted for the flow computation, where a linear domain decomposition scheme is employed to parallelize the flow computation on Cartesian gird via the MPI library[5]. Finite volume method is adopted to discretize the governing equations, and a semi-implicit method for pressure-linked equations (SIMPLE) algorithm[6] is used to decouple velocity and pressure. In the inner iteration, the linearized equations are solved via a strongly implicit procedure (SIP) solver[7] to determine the field variables. A cuboid computational domain is employed in this study. The inflow wind speed varying with height is given in Eq.(9), where U_{in} denotes the inflow velocity, U_{hub} the wind velocity at hub height, z the distance measured from the ground, z_{hub} the hub height and a the ground roughness that is set as 0.1 in this study[8].

$$U_{in} = U_{hub}\left(\frac{z}{z_{hub}}\right)^{a} \tag{9}$$

A zero gradient condition is applied to the field variable at the outflow boundary. A symmetry condition is employed on the lateral boundaries. A no-slip condition is applied to the wall boundary.

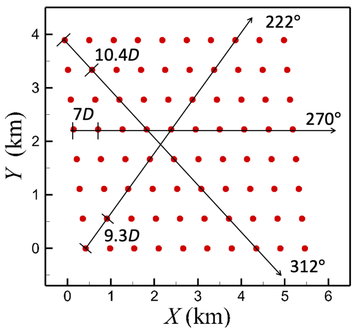

Fig. 1: The definition of wind conditions

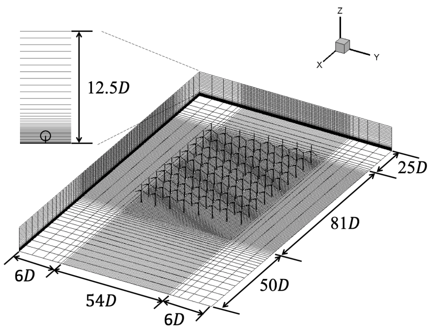

Fig. 2: Computational domain of the target wind farm

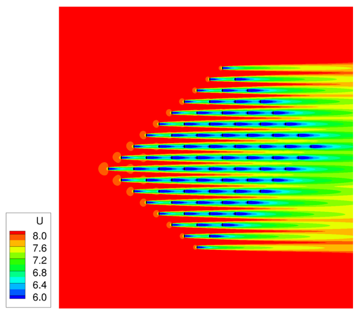

Fig. 3: Wake interaction among wind turbines for $\alpha_w = 312°$

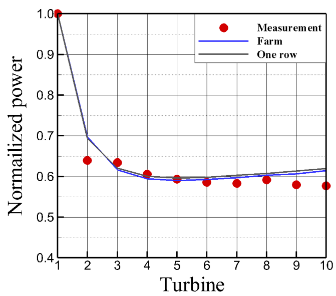

Fig. 4: Normalized wind turbine power for $\alpha_w = 270°$

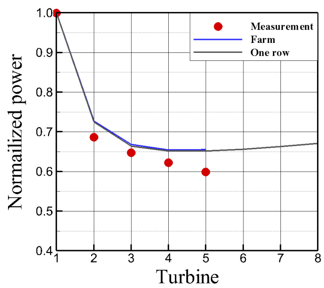

Fig. 5: Normalized wind turbine power for $\alpha_w = 222°$

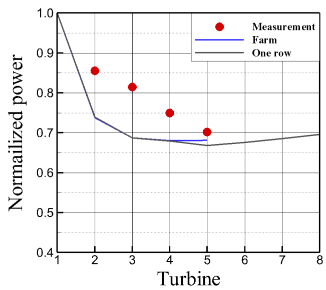

Fig. 6: Normalized wind turbine power for $\alpha_w = 312°$

3 WIND FARM MODELING

The wind turbine layout of the target offshore wind farm is shown in Fig.1, where D denotes the rotor diameter[9]. Eighty V80 2MW wind turbines are installed in the wind farm. The wind condition of 8 m/s at three wind directions, i.e., $\alpha_w = 222°, 270°, 312°$, with the corresponding spacing of $7D$,$9.28D$,$10.4D$ are selected as the modeling cases. Figure 2 depicts a typical grid with about 50 million cells (882×910×59) that is adopted to discretize the computational domain. The wake interaction among wind turbines at the hub height is illustrated in Fig.3 where the downstream wind speed gradually recovers from a velocity deficit resulted by the upstream wind turbines. Figure 4 compares the normalized wind turbine power among the field measurement, the prediction based on a full wind farm modeling and the forecast with a simplified row modeling for $\alpha_w = 270°$ where the power of eight turbines located in the first row is used as the reference value. The numerical prediction indicates the power of downstream rows is only 60% of the reference power that is favorably consistent with the experimental observation. This power behavior clearly the negative wake impact on the downstream wind turbines. Figure 5 and 6 further compare the power performance with large spacing between two consecutive wind turbines. The measurement suggests that wake quickly recovers with the distance between two wind turbines whereas the proposed numerical approach substantially underpredicts the wake recovery, especially for $\alpha_w = 312°$. This discrepancy might be improved for introducing a better mixing model in the downstream area. Additionally, the simplified model is justified to be very computationally economical as well as provide sufficiency accuracy when compared to a full wind farm simulation.

4 CONCLUSIONS

This paper proposes an actuator disk model to investigate the power behavior of wind turbines with wake interaction effect. In the case of $\alpha_w = 222°$ with a wind speed of 8 m/s, the wind turbines of downstream rows only deliver 60% power of the first row where the numerical prediction is well consistent with the measurement. However, large discrepancy is found for $\alpha_w = 312°$. An advanced mixing model is further required in order to deliver sufficient prediction accuracy for wind turbine power with wake interaction effect.

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