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KINETIC CONSISTENT MHD ALGORITHM FOR INCOMPRESSIBLE CONDUCTIVE FLUIDS

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Summary. The aim of this work is to adapt the previously obtained algorithm of magnetogasdynamics for modeling of electrically conductive liquid flows for subsequent application to the study of complex distributed heat transfer systems with electromagnetic propulsion.

1 INTRODUCTION

In [1], based on the use of the complex local Maxwell statistical distribution function and kinetically consistent approach, a computational algorithm is constructed that allows the efficient use of the capabilities of high-performance systems for solution of fundamental and applied problems in magnetogasdynamics.

Many important applied problems require a detailed description of dynamics for incompressible conductive fluids in the electromagnetic fields. The practical applications of this processes is the implementation of energy transfer system with magnetohydrodynamic propulsors in complex heat transfer systems [2].

3 KINETICALLY CONSISTENT APPROACH FOR MODELLING OF AN UNCOMPRESSIBLE CUNDACTIVE FLOWS

The use of a kinetically consistent system of differential equations of magnetogasdynamics for modeling conductive liquids is based on the following physical assumptions. The speed of sound in a liquid is many times higher than the speed of sound in gases. Given that in most technical applications the characteristic velocities don't exceed tens of meters per second, we can say that the condition

$$u \le 0.1\,M \tag{1.1}$$

Is satisfied.

Additional conditions for fulfilling more stringent incompressibility conditions can be specified by the phenomenological equation of state

$$p = p_0 + \beta(\rho - \rho_0), \qquad (1.2)$$

where parameter β reflects the strong influence of pressure changes with a slight change in density, i.e. determines the practical incompressibility of the medium.

In the case of numerical modeling flows characterized by low Mach numbers, a serious problem is the requirement of a sufficiently small time step. In the case of implicit schemes, this requirement is determined by the need to ensure attainability of the required accuracy at which the convergence of the solution slows down significantly. In this regard, explicit schemes are preferable for the implementation of parallel calculations, moreover, the use of hyperbolic-type systems of equations of magnetic hydrodynamics allows a significant reduction in the requirement for stability and, accordingly, the time step [3].

The basis of the research was a kinetically consistent system of differential equations of magnetic gas dynamics of hyperbolic type presented in detail in [4]. For numerical calculations of the incompressible conductive liquid flow, a compact kinetically consistent system of differential equations of magnetic gas dynamics in a compact form is used:

$$\frac{\partial \rho}{\partial t} + \frac{\tau}{2}\frac{\partial^2 \rho}{\partial t^2} + div[(\vec{u} - \vec{w}) = 0 \qquad (1.3)$$

$$\frac{\partial}{\partial t}\rho\vec{u} + \frac{\tau}{2}\frac{\partial^2}{\partial t^2}\rho\vec{u} + div[\rho(\vec{u} - \vec{w}) \times \vec{u} + B_{i,k} + \nabla\left(p + \frac{B^2}{8\pi}\right) = divP_{NS} \qquad (1.4)$$

$$\frac{\partial}{\partial t}E + \frac{\tau}{2}\frac{\partial^2}{\partial t^2}E + div\left[\left(E + p + \frac{B^2}{2}\right)(\vec{u} - \vec{w})\right] = div\, q + div\left(P_{NS}\vec{u}\right) \qquad (1.5)$$

$$\frac{\partial}{\partial t}\vec{B} + \frac{\tau}{2}\frac{\partial^2}{\partial t^2}\vec{B} = rot\left[(\vec{u} - \vec{w}) \times \vec{B}\right] + \nu_m rot\,\vec{B} \qquad (1.6)$$

$$div\,\vec{B} = 0 \qquad (1.7)$$

где: ρ– density, $\vec{u}$– velocity, p– pressure, $w_{i,k} = \frac{1}{\rho}\frac{\partial}{\partial x_i}\left[\left(p + \frac{B^2}{8\pi}\right)\delta_{i,k} + \rho u_i u_k - B_i B_k\right]$, related to solution smoothing at τ, where τ - collisionless characteristic time $\tau = {2\mu}/{p}$, $\tau_m = 2\nu_m\rho \Big/ \left(p + \frac{B^2}{8\pi}\right)$ - magnetodynamic time constant, ν_m - magnetic viscosity, q - heat, $\vec{B}$ - magnetic field.

The numerical solution of the system of equations (1.3) - (1.7) with additional conditions (1.2) was carried out using the three-layer explicit scheme described in [4]. This scheme is ideally adapted to the architecture of computing systems with extra-massive parallelism and is a promising direction for ultra-high-performance parallel computing systems. Asymptotically, the stability of the system under consideration is determined by the condition.

4 NUMERICAL EXPERIMENT

The calculation of the problem of an electrically conductive fluid flow in a flat channel of a magnetodynamic propulsion and its subsequent expansion in the form of a rectangular volume of a heat transfer system, the geometry is shown in Fig. 1.

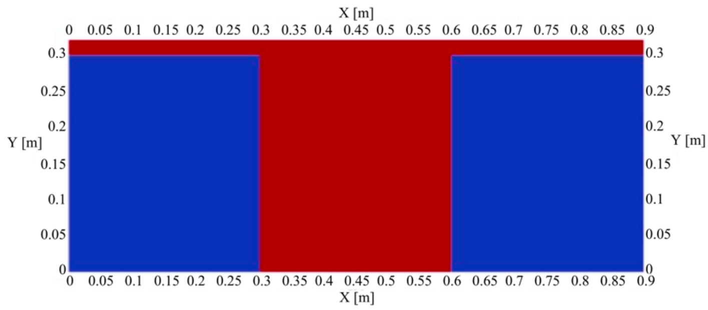

Figure 1: Layout of calculation domain

The beginning part of the input channel 10.0 cm is the region of the electromagnetic propulsion. The electrically conductive liquid Sodium (Na). The physical characteristics were taken from [5]: temperature 600 K, density 874 kg/m^3, adiabatic index 1.2. The parameters of the magnetohydrodynamic propulsion device are determined in such a way that the velocity in the region of the electromagnetic propulsion device is 3.6 *m/ s*.

In fig. Fig. 2 shows the main magnetohydrodynamic observables at the inlet in the steady state. The flow represents a steady laminar flow and the parameters represent: a) the magnetic field strength, b) the velocity in the middle of the channel reaches 2.7 m/s, in Fig. c) the density value in the input channel is presented, the density variation does not exceed 0.2%.

In fig. Fig. 3 shows a 2D projections of the flow dynamics which are characterized by several main stages: at time 0.02 s a transitional regime corresponding to the formation of a stable central vortex, with a shift to the front wall of the cavity due to the reaction of the flow from the back wall of the cavity. At time instant 0.3 s. with a steady state flow, the dynamics stabilizes with the formation of a stable central vortex in the cavity with expected parameters. Input and output flows in the entire time interval remain equivalent with high accuracy, which is typical for incompressible fluid flow.

CONCLUSION

A kinetically consistent model, previously used to model problems associated with the flow of ionized viscous, compressible gases, can also be successfully used to calculate the flows of an electrically conductive incompressible fluid in complex heat transfer systems with magnetohydrodynamic propulsors. The proposed algorithm is important for modeling processes in a number of technical devices on high-performance computing systems.

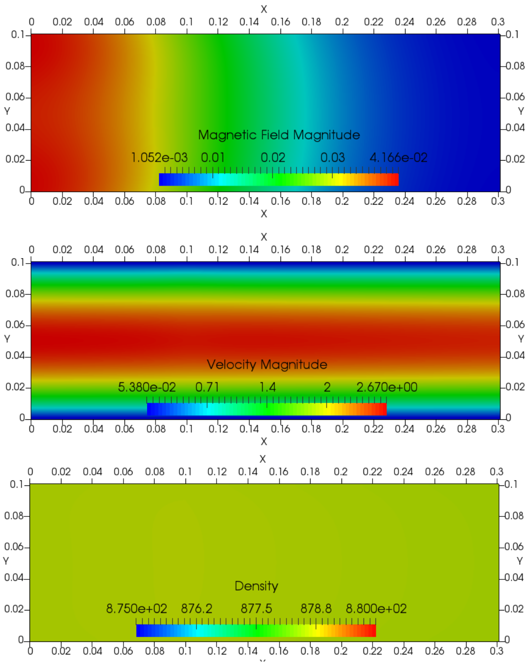

Figure 2: 2D projections of a) magnitude of magnetic field in the region of magnetohydrodynamic propulsion, b) flow velocity of electrically conductive fluid (Na) and c) density of the electrically conductive fluid (Na).

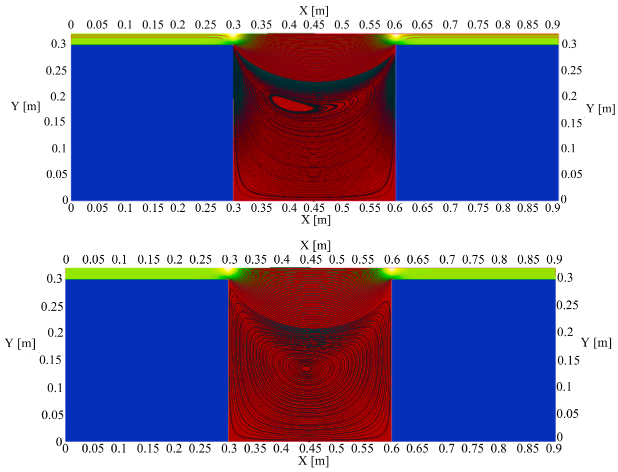

Figure 3: 2D projections of the electrically conductive fluid (Na) flow (density and flow contour) in the region of the cavity in time a) 0.02 *s*. and b) 0.3 *s*.- steady state.

6 AKNOWLEGMENTS

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