ParCFD'2020
32^{nd} International Conference on Parallel Computational Fluid Dynamics
May-11-13 2020, Nice FRANCE

APPLICATION OF THE INVERSE MASS MATRIX APPROACH TO AN EXPLICIT HIGH-ORDER FLUID DYNAMICS CODE

Lucas GASPARINO*, Oriol LEHMKUHL† AND Guillaume C. HOUZEAUX†

* Barcelona Supercomputing Centre
CASE
Carrer de John Maynard Keynes, 30, Barcelona, Spain
e-mail: lucas.gasparino@bsc.es, web page: https://www.bsc.es/gasparino-lucas

† Barcelona Supercomputing Centre
CASE
Carrer de John Maynard Keynes, 30, Barcelona, Spain
e-mail: oriol.lehmkuhl@bsc.es - Web page: https://www.bsc.es/lehmkuhl-oriol/

† Barcelona Supercomputing Centre
CASE
Carrer de John Maynard Keynes, 30, Barcelona, Spain
e-mail: guillaume.houzeaux@bsc.es - Web page: https://www.bsc.es/houzeaux-guillaume/

Key words: Mass matrix, mass lumping

Abstract. This abstract is a summary of our presentation on the effects of the mass matrix on hyperbolic and convection-diffusion systems.

1 INTRODUCTION

In the Finite Elements context, discretization of a set of PDEs containing temporal terms results in the formation of a sparse matrix on the left side of the discrete system, usually called *consistent mass matrix*. Traditionally, explicit temporal schemes substitute this consistent matrix by a diagonal one in a process called *mass lumping*, in order to avoid solving a costly linear system.

Although very efficient performance-wise, for hyperbolic and convection-dominated problems this procedure leads to solutions that are more oscillatory compared to their consistent mass counterparts, especially when non-linear (high-order) elements such as the 27-node hexahedron are used.

Given the dilemma of cost-efficiency against accuracy of the solution, we propose an implementation of Guermond's *et. al.* method for approximating the *inverse* of the consistent mass matrix, adapted to act as a low-cost iterative solver, as an effective middle ground, since it requires very few iterations (1 to 4) to mimic the effect of using a full consistent mass solver.

2 METHOD

According to Guermond, the consistent mass matrix can be approximated by the series:

$$M_c^{-1} = \left[\sum_{n=0}^{\infty} A^n\right] \overline{M}^{-1} \tag{1}$$

where $\overline{M}$ is an easily invertible matrix and A^n are sucessive matrix-matrix multiplications. Matrix A can be obtained by a left, right or symmetric factorization of M_c; since all three lead to the same result in this particular case, we choose the right factorization, that is:

$$A = A_r = \overline{M}^{-1}\left(\overline{M} - M_c\right) \tag{2}$$

The obvious choice is for $\overline{M}$ to be diagonal; we propose to use diagonal-scale lumping to obtain $\overline{M}$, that is:

$$diag(\overline{M}_e) = \alpha * diag(M_e^c) \tag{3}$$

$$\alpha = V_e/tr[M_e^c] \tag{4}$$

$$\overline{M} = \mathbf{A}_{e=1}^{N_e}[\overline{M}_e] \tag{5}$$

In the set above, V_e is the element volume (area for 2D, length for 1D), and $\mathbf{A}_{e=1}^{N_e}$ is the element assembly operation defined by T. J. R. Hughes. Notice that both the right factor and the inverse matrix are obtained already assembled.

Our application of this method consists in truncating the series at a fixed number of terms and substituting the costly matrix-matrix multiplications by sucessive matrix-vector products, implying that only the vector is modified at any time. This avoids parallelization issues, as well as storage of increasingly dense A^n matrices. The method is convergent as long as $\rho(A) < 1$, which is assured when obtaining $\overline{M}$ by diagonal scaling.

3 EXPECTED CONCLUSIONS

By the time of this presentation, we expect to have tested this method in at least three different scenarios:

- A Re=3900 incompressible flow over a single cylinder;
- A Re=16000 incompressible thermal flow over two cylinders in tandem;
- A Re=16000 Low-Mach flow over two cylinders in tandem;

All these cases will be tested with both linear and quadratic elements, where we expect to obtain more smooth solutions with little to no extra cost to the substeps of the Runge-Kutta solver when using the approximate inverse. Fig. 1 provides an example of the expected results for the Re=3900 case with a Q2 mesh. We will also present 1D results that demonstrate the impact of mass lumping in a more textbook scenario, and which allows us to experiment with different kinds of stabilization procedures. Fig. 2 provides examples for both pure Galerkin Rk4 and entropy viscosity RK4 solutions.

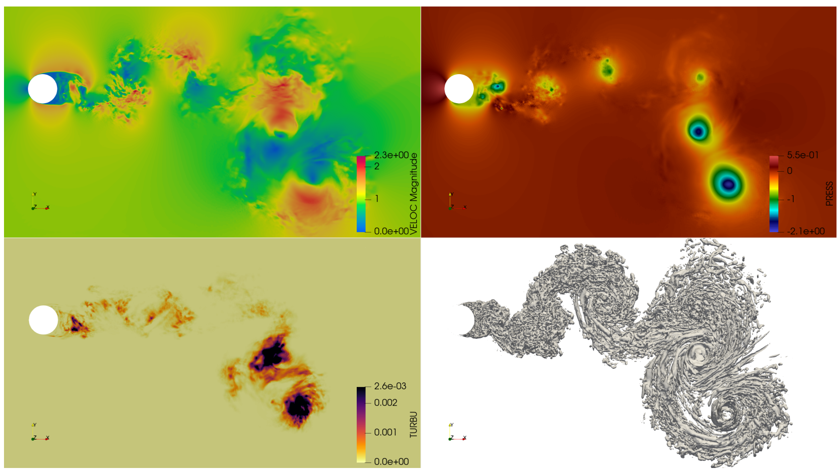

Figure 1: Starting from top left, in clockwise direction: velocity magnitude, pressure, Q-criterion and subgrid turbulent viscosity. Obtained using Q2 elements, RK4 time-stepping and 2 iterations of the inverse mass approximation.

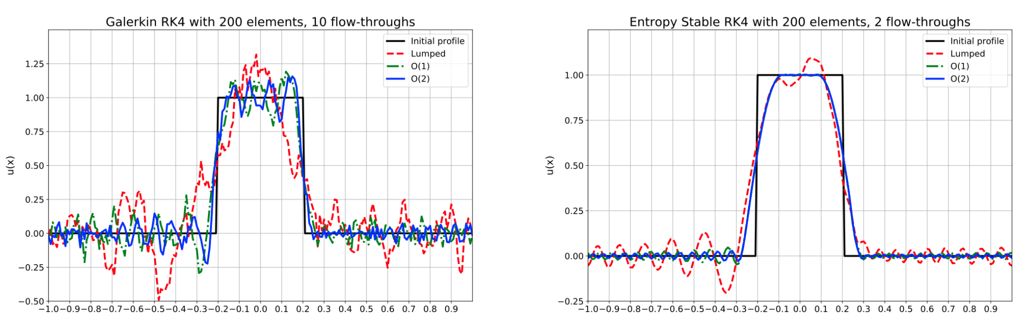

Figure 2: Comparison of lumped and consistent mass solutions for a scalar hyperbolic equation. On the right: Galerkin RK4; On the left: Entropy stable RK4

REFERENCES

[1] Guermond, J.L. and Pasquetti, R. A correction technique for the dispersive effects of mass lumping for transport problems.