A MACHINE LEARNING APPROACH TO CYLINDER FLOW COMPUTATIONS

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Summary. We apply Machine Learning in the form of Reservoir Computing approach to Proper Orthogonal Decomposition (POD) models of 3D cylinder flow.

ABSTRACT

We apply Machine Learning in the form of Reservoir Computing[1,2] approach to Proper Orthogonal Decomposition (POD)[3,4] models of 3D cylinder flow. POD modal decomposition is used for dimension reduction and model generation of low-order models that approximate the dynamics of the full system. The full incompressible Navier-Stokes model is the spectral-element solver Nek5000[5,6], set up to simulate 3D cylinder flow with a 1x20 aspect ratio circular cylinder. Periodic boundary conditions are applied at the cylinder ends. We performa the simulation at Re=200, just beyond the onset of 3D instability but well below onset of additional 3D instabilities to the 2D state. Here we find that the flow is chaotic (see Figure 1). The cylinder is in the regime between $Re_2 \approx 188.5$ where the 2D wake is unstable to 3D disturbances while at $Re'_2 \approx 259$, additional 3D instabilities occur[7,8].

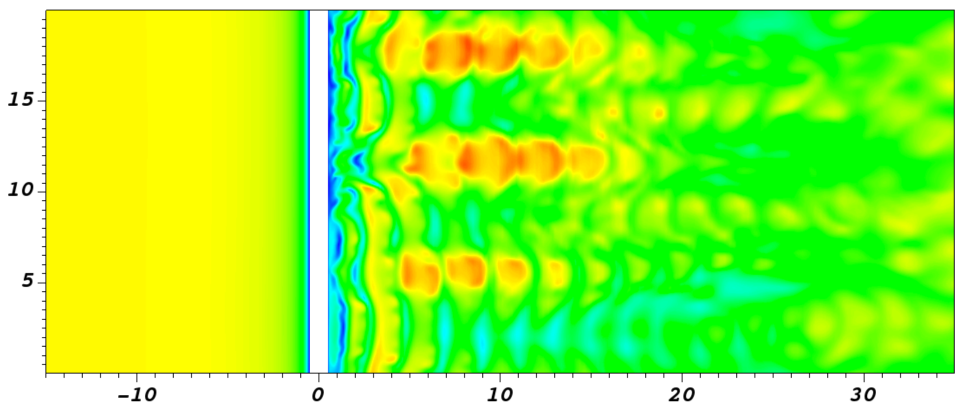

Figure 3: 3D cylinder shown in transverse view.

Reservoir Computing approach is a recurrent neural net (RNN) where the connectivity of the reservoir neurons is represented by values drawn from a uniform random distribution[9]. The output of the autonomous reservoir v(t), gives the predicted value u(t) for t > 0. Because the output can be computed in one shot it is much faster than training general RNNs.

$$\mathbf{r}(t+\Delta t) = \tanh[\mathbf{A}\mathbf{r}(t) + \mathbf{W}_{in}\mathbf{u}(t)],$$
$$\mathbf{v}(t+\Delta t) = \mathbf{W}_{out}(\mathbf{r}(t+\Delta t), \mathbf{P}),$$

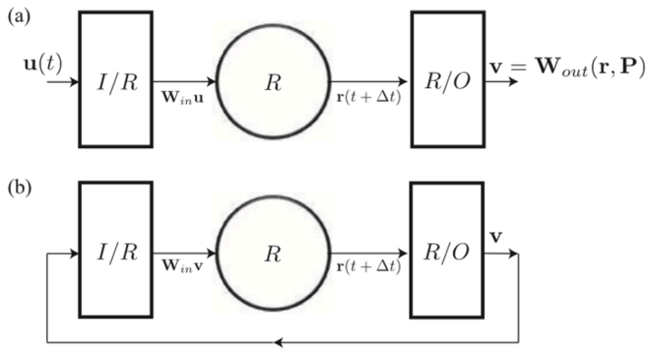

Figure 1: RC approach uses the (a) training and (b) prediction. After [1].

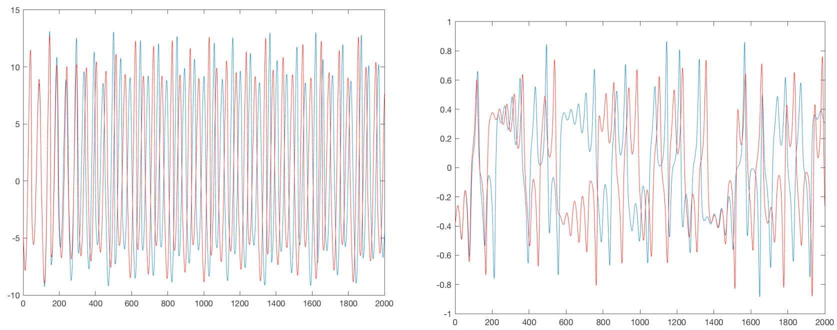

Figure 2: Left, Lorenz & Rossler give similar results, about 2 – 3 cycles. Subsequently, the small deviations send the trajectory into the basin of the other attractor as in right figure. Right, Lorenz showing this explicitly.

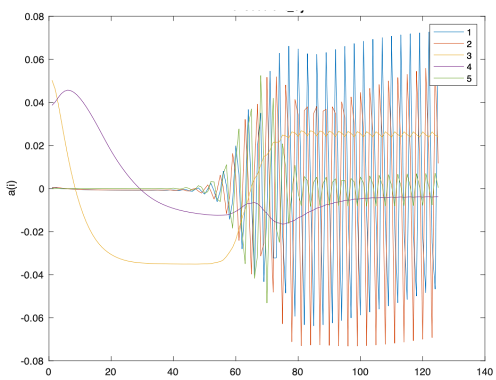

Figure 4: Time evolution of the POD Modes 1-5.

The POD model system is

$$\mathbf{u}(t,\mathbf{x}) = \mathbf{u}_m + \sum_{i=1}^{M} a_i(t)\mathbf{\Phi}_i(\mathbf{x})$$

where the evolution equations are,

$$\frac{da_k}{dt} = A_k + \sum_{i=1}^{N} B_{ki}a_i + \sum_{i=1}^{N}\sum_{j=1}^{N} C_{kij}a_ia_j,$$

with coefficients given by the projection onto the NS system[4]. Figure 4 shows an integration of the model. The full paper gives the details of the RC procedure and the application to the model system with various truncation levels. It points to the prediction using the RC and POD and combining them as a predator-corrector set up for better prediction of the full Navier Stokes system. The full paper describes a basis for applying this to engineering calculations such as lift-drag curves, where the dynamic computations provide improved efficacy. Parallel computation results are emphasized.

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