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SPACE-TIME PARALLEL CFD ALGORITHMS WITH ADAPTIVITY FOR COMPRESSIBLE FLOWS

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Abstract. The present study applies a space-time parallel CFD algorithm with adaptivity to solve fluid dynamical systems governed by the compressible Navier-Stokes equations. The space-time parallelism is achieved by multigrid reduction-in-time while the adaptivity is realized using adaptive mesh refinement in space with subcycling in time. Verified and validated, the space-time parallel algorithm is demonstrated by solving compressible flows with convection, diffusion, and reaction.

1 INTRODUCTION

The birth of supercomputers fundamentally changed the design and implementation of CFD algorithms, along with their engineering applications, notably, to aerospace. Parallel CFD algorithms allow for solving fluid dynamics problems with the increasing complexity and high-confidence prediction of flow physics, and the aerospace industry has seen a 50% reduction in wind tunnel testing since 1980's. Nevertheless, CFD has been on a plateau during the past decade [9]. To adequately solve vortex dominated and transitional flows at high-Reynolds number, particularly with the presence of solid surfaces due to complex geometries, high-order large eddy simulation algorithms are necessary. Massively parallel implementation of these algorithms must be explored in both spatial and temporal domains to benefit from the growing computer power with unprecedented evolution, such as individual processor speeds have stalled while concurrency has increased (See Fig. 1).

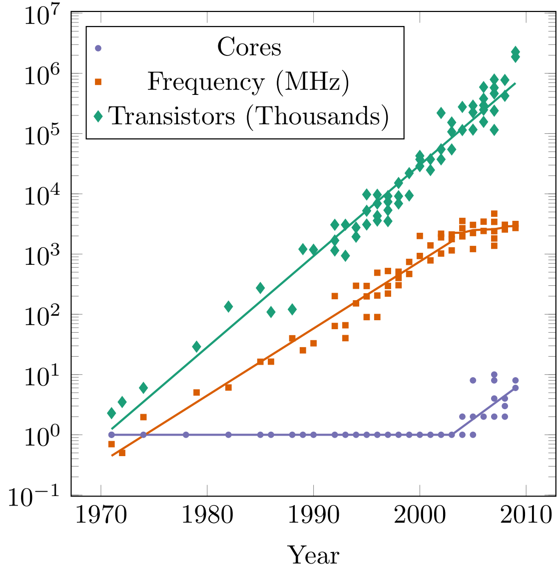
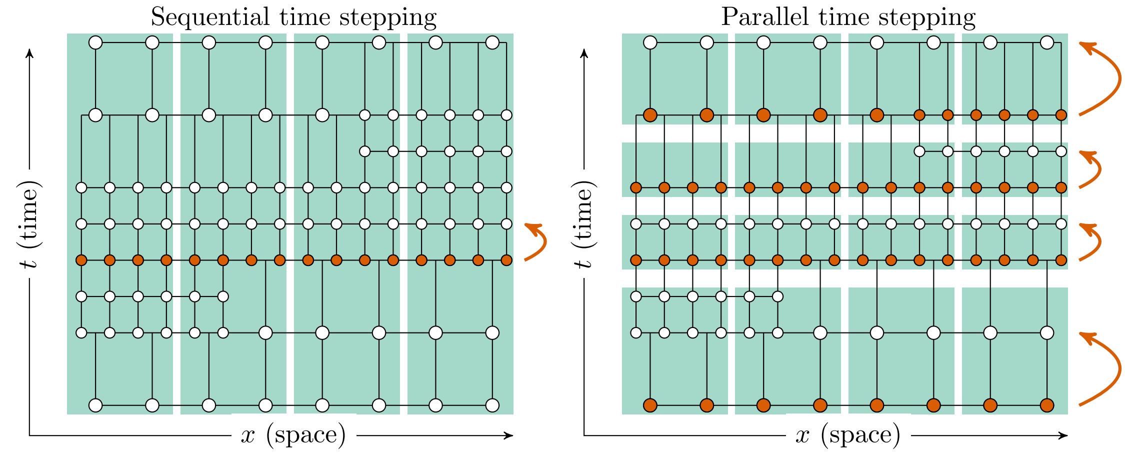


Figure 1: Individual processor speeds have stalled while concurrency has increased. Adapted from Yelick[12]

The diagrams in Fig. 2 portray two parallel algorithms with adaptive mesh refinement (AMR); Fig. 2a shows the spatial parallelism while time stepping is sequential and Fig. 2b shows parallelization in space and time, the present algorithm. The present adaptive parallel space-time algorithm is built upon several software packages developed by the computational mathematics and CFD communities. Specifically, XBraid [2], a non-intrusive open-source implementation of multigrid reduction in time (MGRIT), enables the temporal parallelism. Chombo [1], an open-source framework for patch-based structured AMR, provides refinement in space and time. The CFD application code, Chord [8, 7], is a fourth-order accurate finite-volume method solving compressible flows, with chemical reactions if combustion is considered, and the solution is advanced in time using the standard four-stage Runge-Kutta method. Further information on the algorithmic implementation, its verification and validation, along with the speedups can be found in our recent work [4, 3, 5].



(a) Time-sequential

(b) Time-parallel

Figure 2: Parallelization strategy for a time-sequential algorithm and a time-parallel algorithm.

2 Preliminary Results

While the parallel space-time algorithm has been successfully demonstrated for fluid dynamics problems dominated by diffusion physics or periodicity in time [4, 3, 5], convective physics appears to challenge the convergence rate of multigrid methods [6] which are used for the time parallelization. The present study is intended to demonstrate that the algorithm is effective for solving multi-physics fluid dynamics problems. First, we perform a verification of the algorithm using the method of manufactured solutions. Following the methodology described by Salari and Knupp[11], a solution is manufactured (given in the supplementary file) to the compressible Navier-Stokes equations including continuity, momentum, and energy. With Maxima [10], the source terms, as well as the initial and boundary conditions are calculated. Trigonometric functions create a solution field that varies spatially, and a Gaussian function perturbs the amplitude of the trigonometric functions in time. This creates solution gradients that vary over space and time, allowing for the AMR application. The space-time grid with AMR is exhibited by Fig. 3 where mesh refining and coarsening are executed in response to the physics-based criteria on the spatial gradients and the rate of change of the solution variables. The grid convergence rate of the space-time parallel algorithm is examined and compared to the time-sequential, space-parallel algorithm in Figs. 4 and 5 for mass and x-momentum, respectively, verifying the fourth-order solution accuracy of the underlying CFD solver (for all solution variables, not shown here). Furthermore, the figure confirms that the integrated algorithm (Chord/Braid) reproduces the fourth-order accuracy when executing in the sequential time stepping. Figure 6 compares the mass through the center-line in the time-parallel to the exact solution, showing the close agreement. Conservation is preserved by both algorithms. Figure 7 measures the multigrid convergence on a 4-level grid. The slow convergence rate is far from ideal, and the investigation is being

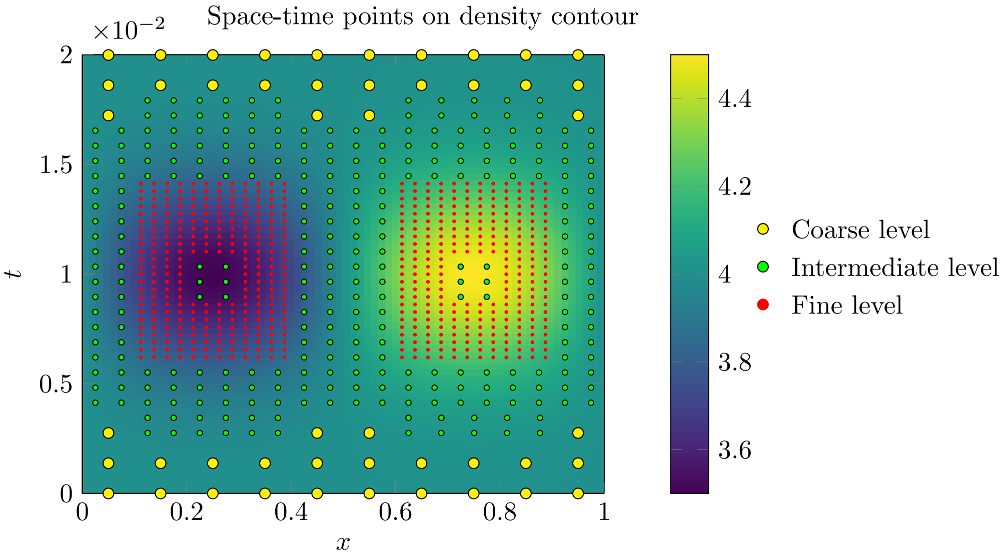


Figure 3: A representation of the space-time mesh generated for the case.

carried out. The verified and validated space-time parallel algorithm will be applied to solve CFD problems with convection, diffusion, and reaction. At the conference, we will present detailed findings, including the speedups of the space-time parallel algorithm over the conventional space-parallel algorithm.

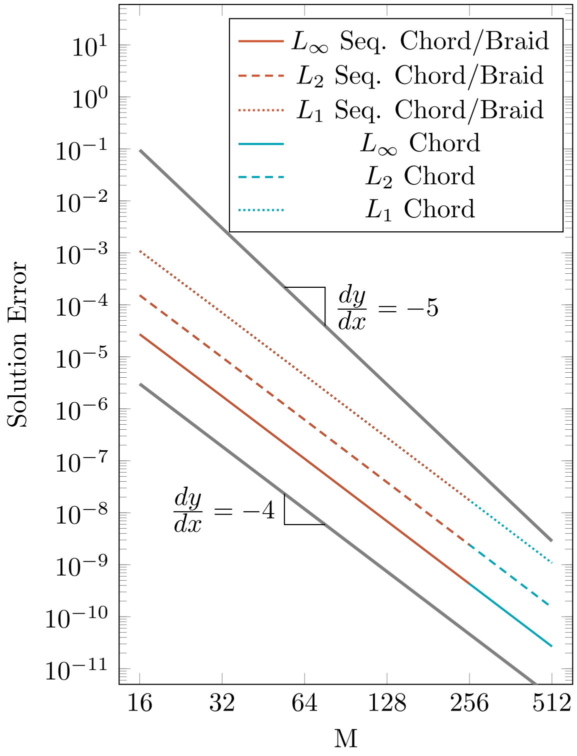


Figure 4: Grid convergence rate of ρ

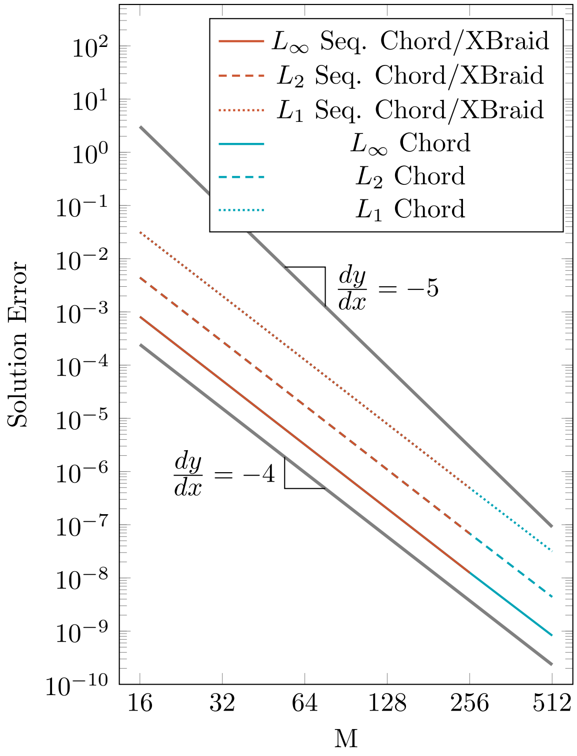


Figure 5: Grid convergence rate of ρu.

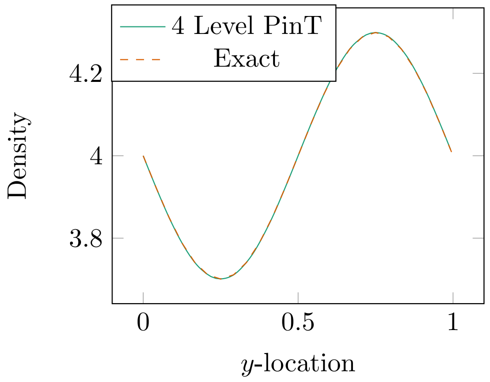


Figure 6: A comparison of the 4 level parallel-in-time solution to the exact solution at x=0.5 and t_n=640.

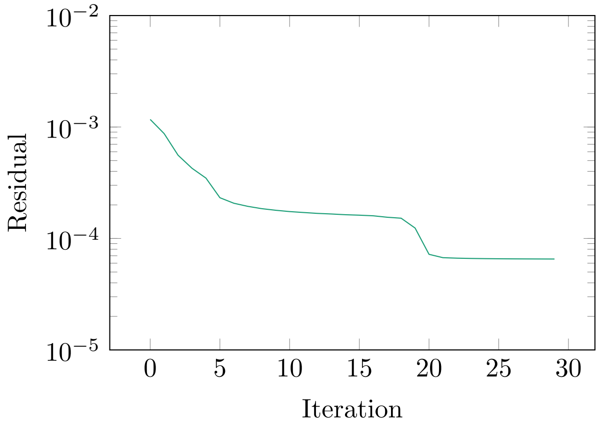


Figure 7: Multigrid convergence.

Acknowledgments

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