

PARALLEL MESH REZONING FOR 2D ARBITRARY LAGRANGIAN-EULERIAN FLUID COMPUTATION

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Summary. Mesh rezoning is one of the most important steps in Arbitrary Lagrangian-Eulerian (ALE) methods in computational fluid dynamics, which a new mesh of high quality is defined based on the old Lagrangian mesh. Here we study a typical winslow smoothing method on two-dimensional unstructured meshes. It is a Gauss-Seidel style computation with small parallel granularity. We firstly evaluate the arithmetic intensity of winslow smoothing method, and provide two parallel strategies with different usage. One is the parallel multi-color ordering algorithm used for a little iterations, the other is parallel pipeline algorithm for a large amount of iterations. We also discuss the parallel performance results obtained on a parallel computer when these methods are applied for a large scale ALE fluid application with a hundred billion grids.

1 INTRODUCTION

Traditionally, fluid dynamics is classified into Lagrangian and Eulerian methods. In Lagrangian methods, the computational mesh travels with the fluid, where the computation can only take over some time before severe mesh distortion. In Euler methods, the mesh is fixed, where the sharp resolution of interfaces or free surfaces is lost.

Arbitrary Lagrangian-Euler (ALE) methods have been developed to overcome the above limitations and combine the features of both the Lagrangian and Eulerian methods. These methods typically include three phases: a Lagrangian phase in which the mesh moves in the Lagrangian manner, a mesh rezoning phase in which a new mesh of highly quality is defined, and then a remapping phase in which all the computational quantities defined on the mesh are transferred from the Lagrangian mesh to the new mesh. It is with the mesh rezoning phase that we will be concerned in this paper.

The winslow smoother is a typical method used for the mesh rezoning in two-dimensional unstructured mesh applications. The new position of a vertex of the mesh is determined by its neighboring eight vertices and itself in two-dimensional case. It is a Gauss-Seidel style computation with less small parallel granularity, since there is a computational dependency

between adjacent vertices because one vertex needs to be optimized after the other. In the following, we call this method as 2D nine-point Gauss-Seidel style scheme.

The novel contributions of this paper are to evaluate the computational intensity of the winslow smoothing method, introduce two parallel algorithmic strategies for its different usage in ALE fluid computation, and to highlight implementation to achieving good scaling.

2 ARITHMETIC INTENSITY OF WINSLOW SMOOTHING

In Algorithm 1, the sequential algorithm of the 2D nine-point Gauss-Seidel style scheme can be seen. The inputs of the sequential algorithm are the followings: M is the rezoning mesh, $maxIter$ is the maximum number of the mesh rezoning iterations, $Neighbors$ is the set of vertices connected to the vertex v , X is the initial position of the vertex v , X' is its position after an optimization, Q measures the quality of the optimized mesh. Here $solver()$ is to solve the winslow equation. The output of the algorithm is a rezoned mesh M , whose minimum quality must be larger than a user-specified threshold θ . This algorithm iterates over all the mesh vertices in some order and adjusts at each step the coordinates of the vertices.

Algorithm 1: Sequential algorithm of the 2D nine-point Gauss-Seidel style scheme.

```
res = 0
iter = 0
While res <  $\theta$  and iter < maxIter DO
  For each vertex  $v \in M$  do
     $X' = solver(X, Neighb)$ 
  End do
  Res = quality(M)
  Iter = iter + 1
End do
```

In the sequential Algorithm 1, there is a computational dependency between adjacent vertices because one vertex needs to be optimized after the other. The computation ordering prevent two adjacent vertices from being simultaneously rezoned on different processors when the sequential algorithm is parallelized.

We evaluated the arithmetic intensity of 2D nine-point Gauss-Seidel style scheme. Here the arithmetic intensity is defined as the ratio between the numbers of floating point operations and the main memory traffic. We give two arithmetic intensity of this method based on a grid cell and on a block including many cells connected with each other respectively. Its arithmetic intensity is 0.32~0.56. The analysis show that the 2D nine-point Gauss-Seidel style scheme is a memory-bounded method and the ratio between computation and memory access is too low. Its performance is more dependent on the memory access than computation speed.

3 TWO PARALLEL STRATEGIES FOR MESH REZONING

For different usage of the mesh rezoning in real application, we provides two parallel strategies for the 2D nine-point Gauss-Seidel style scheme respectively. One is a parallel

multi-color ordering algorithm when the number of mesh rezoning iterations is small, the other is a parallel pipeline algorithm when the number of mesh rezoning iteration is achieved by several hundreds and thousands. These parallel algorithms will be introduced in the following.

3.1 Multi-color ordering parallel algorithm

For the case that the number of mesh rezoning iteration is small, a multi-color ordering parallel algorithm is provided. Firstly, each vertex of the old Lagrangian mesh is colored, and all the adjacent vertices have different colors. Now all the vertices are classified into several groups with the different color. The vertices with the same color have no computational dependency, so they can be parallel optimized. The previous Gauss-Seidel style computation is transformed into several Jacobi updating.

3.2 Parallel pipeline algorithm

After evaluating the arithmetic intensity of winslow smoothing method, we know that it is a memory-bounded scheme, and can not get high performance by only parallel partition the mesh. It should partition the iteration space to achieve high performance.

For the case that the number of mesh rezoning iteration is large, we provide the parallel pipeline algorithm for the method. Here, domain decomposition is not only occurred in the mesh, but also in the iteration space. Unlike the traditional parallel method, one iteration and a part of mesh are grouped together as a basic computation unit and are mapped into a process, the same iteration and the adjacent part of mesh are also grouped and mapped into another process, and so on. It forms a parallel pipeline, in which each process only process one step on the pipeline.

4 NUMERICAL EXPERIMENTS

4.1 Performance of parallel multi-color ordering algorithm

We run these parallel algorithms in the same parallel computer. When the iteration number is 10 and the mesh is a hundred billion grids, the parallel efficiency of the multi-color ordering algorithm can be 93.8%, 90.12%, 90.12%, and 89.96% on 128, 256, 512, and 1024 processor kernels respectively.

4.2 Performance of parallel pipeline algorithm

Figure 1 gives the parallel efficiency of the pipeline algorithm when running 200 iterations on a billion grid cells. The parallel efficiency is about 60% when running on 128 processor kernels. Figure 2 compares the parallel efficiency when running different iteration numbers. It shows that larger iteration numbers can achieve better performance. When running on 64 processor kernels, the execution time of the case with iter=200 is decreased 60% compared to the case with iter=10.

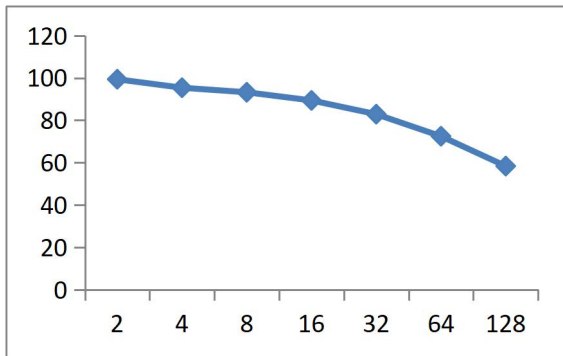


Fig 1. Parallel efficiency of parallel pipeline algorithm when running with a billion grids and 200 iterations.

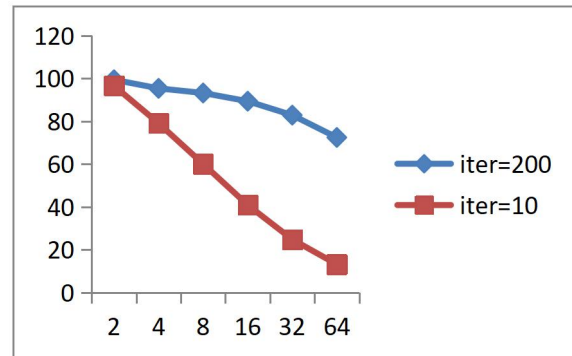


Figure 2. Parallel efficiency of parallel pipeline algorithm when running with a billion grids and different iteration numbers

5 CONCLUSIONS

- Mesh rezoning phase is important in ALE fluid computations. Winslow smoothing is a typical mesh rezoning method. By evaluating the arithmetic intensity of this method based on a cell and on a block, we show that it is obviously a memory-bounded method in which the ratio between computation and memory access is low.
- Winslow smoothing is also a typical Gauss-Seidel style computation where the strong computational dependency between adjacent cells. Two parallel strategies are provided to the winslow smoothing method with different usage. One is the multi-color ordering parallel algorithm when the iteration number is small, the other is the parallel pipeline algorithm for larger iteration number cases. We applied them for the parallel fluid computation with billion grids. These similar parallel strategies can also be applied for more Gauss-Seidel style computations.

REFERENCES

- [1] John K. Dukowicz and John W. Kodis. Taylor, *Accurate conservative remapping(rezoning) for arbitrary lagrangian-eulerian computations*. SIAM J. SCI. STAT. COMPUT. Vol.8, No. 3, May 1987
- [2] Loyce M. Adams and Harry F. Jordan, *Is SOR color-blind?* SIAM J. SCI. STAT. COMPUT. Vol.7, No. 2, April 1986.
- [3] G.J. Gorman, J. Southern, P.E. Farrell, M.D. Piggott, and et al., *Hybrid OpenMP/MPI anisotropic mesh smoothing*, Procedia Computer Science 9 (2012) 1513-1522.
- [4] Partick M. Knupp, *Winslow smoothing on two-dimensional unstructured meshes*. Engineering with Computers 15, 263-268 (1999).