ParCFD'2020
32nd International Conference on Parallel Computational Fluid Dynamics
May 11-13 2020, Nice FRANCE

# ARTIFICIAL COMPRESSIBILITY METHOD WITH BULK VISCOSITY TERM FOR AN UNSTEADY INCOMPRESSIBLE FLOW SIMULATION

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**Key words:** Artificial compressibility method, Bulk viscosity, Unsteady incompressible flow.

**Summary.** We introduced bulk viscosity term into the artificial compressibility method (ACM) for unsteady incompressible flow simulation to improve the efficiency of ACM. The stability, accuracy, and computational effort were investigated to clarify the bulk viscosity effects. ACM with bulk viscosity term (BTACM) could improve the stability and the accuracy of the ACM simulation especially for the coarse grid case. As a result, BTACM is 5.94 times faster than ACM within a permissible error of 5 percent.

## 1 INTRODUCTION

In last two decades, to solve massive and complicated engineering problems in fluid dynamics, the development of high speed digital computers has been accelerated. Currently, scientific computers shift toward parallel computation using many-core CPUs and general-purpose processing on graphic processor units. To take advantage of these parallel computational architectures, algorithms that are suitable (i.e., highly efficient and highly scalable) for parallel computation are required. The ACM proposed by Chorin[1] adds a pressure time derivative term into the continuity equation by assuming the pseudo-compressibility of the fluid; thus, the pressure can be solved locally by the time evolution equation without solving the Poisson equation. Therefore, ACM algorithm is noted by the suitability for parallel computation. However, the ACM solution contains more or less an artificial sound wave due to the pseudo-compressibility of the fluid, which sometimes prevents the solution from converging. To dump the pressure fluctuation, the bulk viscosity was introduced into the momentum equation and showed that the bulk viscosity term enhances the dissipation of the sound waves (Ramshaw and Mousseau[2], Ohwada et al.[3]).

Despite the abovementioned studies, the effect of the bulk viscosity on the ACM simulation, especially for the unsteady flow, has not been sufficiently studied. In this study, we investigated the effect of the bulk viscosity on the stability, accuracy, and computational effort of the ACM simulation for unsteady incompressible flow simulation.

## 2 NUMERICAL METHODS

In the ACM with bulk viscosity term (BTACM) were used in this study, the governing equations of BTACM are given as following equations (1) and (2)

$$\frac{dp}{dt} = -\beta\left(\frac{\partial u_i}{\partial x_i}\right), \qquad \beta = \frac{1}{Ma^2}, \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_j\frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{\mathrm{Re}}\frac{\partial^2 u_i}{\partial {x_j}^2} + A\frac{\partial}{\partial x_i}\left(\frac{\partial u_k}{\partial x_k}\right), \tag{2}$$

where, $u_i$ is the fluid velocity component in the i-direction, and $p$ is the pressure. The pressure is explicitly calculated from this time evolution equation (1). $\beta$ is the pseudo-compressible parameter which has the relationship with the Mach number, $Ma$, Re is the Reynolds number. The only difference of BTACM from original ACM is the bulk viscosity term which introduced into the third term of right hand side in momentum equation (2). $A$ in equation (2) is the dimensionless bulk viscosity coefficient written as follows,

$$A = \frac{1}{\mathrm{Re}}\left(\frac{\eta}{\mu} + \frac{1}{3}\right), \tag{3}$$

where the $\eta$ is the bulk viscosity, $\mu$ is the viscosity. Since the bulk viscosity term includes the divergence of the velocity, it becomes zero when the incompressibility condition is completely satisfied. Actually in the ACM simulation, the small compressibility is permissible. Therefore, the bulk viscosity term has a non-zero value, and activates to suppress the pressure fluctuation. $A$ is free parameter for the incompressible flow simulation, so it was changed in the range $A \leq 0.2$ in this study.

The discretization of equations (1) and (2) was performed by the finite difference method. 2nd and 4th order central difference approximations were used for the space derivative terms. For the time integration, the 4th order 4-step Runge-Kutta scheme by Jameson and Baker[4] was used. Calculation was carried out on the uniform grid with the regular arrangement of velocities and pressure.

## 3 RESULTS AND DISCUSSION

### 3.1 Stability, accuracy and efficiency of BTACM

To investigate the stability, accuracy and efficiency of BTACM for an unsteady flow problem, a doubly periodic shear layer (Minion and Brouny[5]) at $Re$ = 10,000 was calculated using $\beta = 100$ as test problem.

First of all, the stability of ACM and BTACM were compared. When the grid resolution is less than 256 × 256, ACM cannot obtain a stable solution. However, the use of BTACM can obtain the converging solution in such coarse grid case. Specifically, the use of $A$ greater than $5.0 \times 10^{-2}$ was able to obtain the solution even for the 32 × 32 grid. Thus, the addition of the bulk viscosity term can enhance the stability from original ACM.

The validation of computed flow patterns by BTACM was performed by the vorticity distributions in Fig.1. ACM simulation using coarse grid generally occurs spurious vortices, whereas, the BTACM simulation can solve the flow field without spurious vortices as shown in Fig.1. The $L_2$ errors from the reference data (pseudo-spectral method, 1024 × 1024) shown in Fig. 2 show that when the grid resolution is less than 128 (i.e., in under-resolved cases), the bulk viscosity can improve in accuracy of solution. These results indicate that BTACM is effective for the under-resolved simulation.

To investigate computational efficiency of BTACM, CPU time of BTACM was compared with that of ACM. Since the computational effort for additional calculation of bulk viscosity term is very small, furthermore, BTACM can obtain the solution with lower grid resolution than ACM, BTACM was able to calculate about 5.94 times faster than original ACM within permissible error of 5 percent for reference solution.

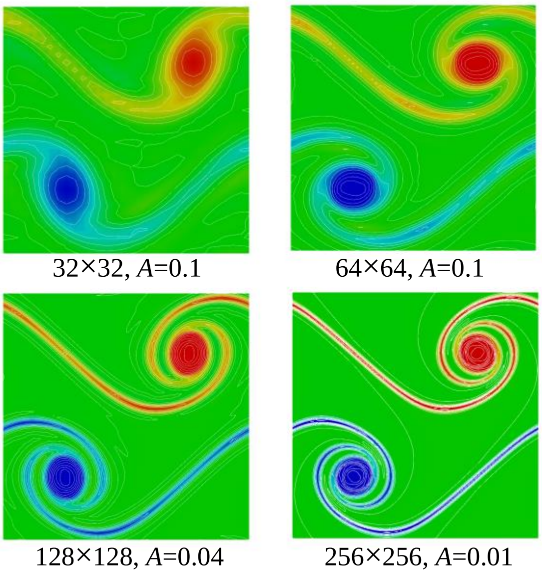



Figure 1:Vorticity distributions (4th order discretization)

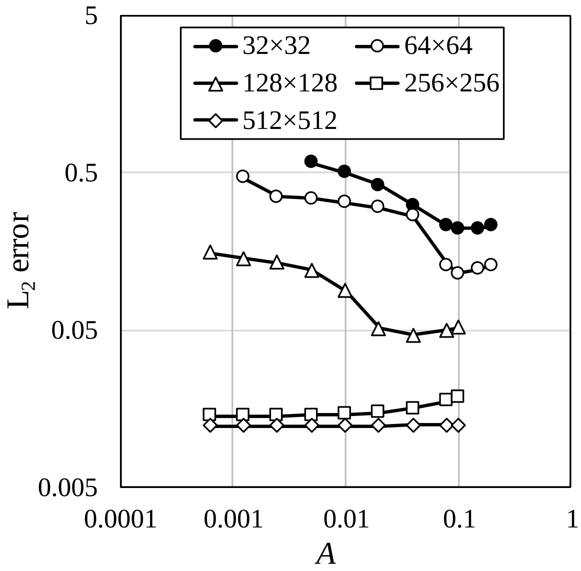
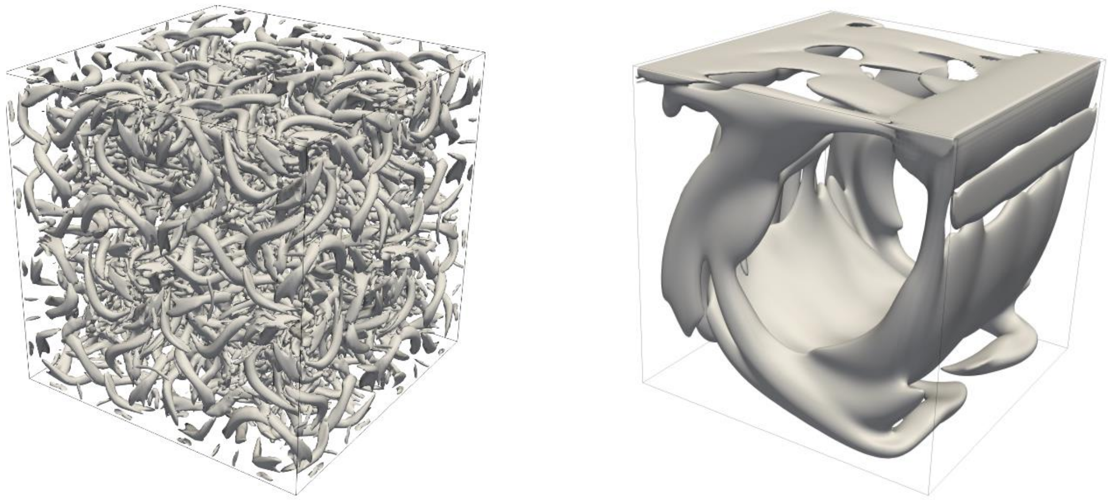



Figure 2: $L_2$ error vs. bulk-viscosity coefficient $A$

### 3.2 Application of BTACM to three-dimensional flow using parallel computation

To demonstrate the applicability of BTACM to parallel computation, three-dimensional decaying isotropic turbulence and three-dimensional lid-driven cavity flow were calculated using BTACM with 2nd order discretization and grid resolution of $128^3$ on GP-GPU platform. As shown in Fig.3, the three-dimensional vortex structure of decaying isotropic turbulence and lid-driven cavity flow were solved reasonably in such low grid resolution.



$Re$=4000, $\beta$=1600, $A$=1.6×10$^{-3}$, $Q$=500 $Re$=3200, $\beta$=100, $A$=3.1×10$^{-4}$, $Q$=0.2

Figure 3: Iso-surface of $Q$-criterion (left: isotropic decaying turbulence, right: lid-driven cavity)

## 4 CONCLUTIONS

In this study, to clarify the effect of the bulk viscosity on the ACM simulation for unsteady incompressible flow, we investigated the stability, the accuracy, and the efficiency of the BTACM simulation. BTACM improved the stability, and the accuracy of original ACM for the under-resolved simulation. Furthermore, BTACM improved the efficiency of the simulation. The speed up ratios of BTACM for original ACM were 5.94 for the permissible $L_2$ error of 0.05.

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