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A NOVEL METHOD FOR MAGNETOHYDRODYNAMIC SIMULATIONS AND ITS APPLICATIONS IN ASTROPHYSICS AND COSMOLOGY ON HIGH PERFORMANCE COMPUTATIONAL SYTEMS

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Abstract. In this work we present a novel method to carry out magnetohydrodynamic simulations using kinetic consistent schemes for the solution of the Boltzmann Equation. This method is then applied to specific problems in cosmology and astrophysics.

Magnetic fields are one of the most important phenomena in science and engineering, as they are present on almost every scale in nature, ranging from atomic magnetic moments to the intergalactic scales, and are used in applications ranging from Magnetic Resonance Imaging to nuclear fusion.

In this work we first present a novel powerful method for high performance magnetohydrodynamic (MHD) calculations which is based on kinetic schemes. In particular, using it, it is possible to derive the MHD equations directly from the Boltzmann Equation without the necessity of an *ad hoc* introduction of terms related to electromagnetic interactions.

The fluid dynamics quantities and equations can be derived from the distribution function f which is governed by the Boltzmann equation which, without external forces, reads

$$\frac{\partial f(\mathbf{x}, \boldsymbol{\xi}, t)}{\partial t} + \boldsymbol{\xi} \cdot \nabla f(\mathbf{x}, \boldsymbol{\xi}, t) = \mathcal{C} f(\mathbf{x}, \boldsymbol{\xi}, t)\,, \tag{1}$$

where $\mathbf{x}$ and $\boldsymbol{\xi}$ are the position and velocity vector, respectively, and $\mathcal{C}f$ is the collision integral.

The relevant quantities can be obtained by taking the so-called moments of $f(\mathbf{x}, \boldsymbol{\xi}, t)$, i.e. by calculating the integral

$$\mathcal{M}_i f(\mathbf{x}, \boldsymbol{\xi}, t) = \iiint \phi_i(\boldsymbol{\xi}) f(\mathbf{x}, \boldsymbol{\xi}, t) \mathrm{d}^3\xi \,, \tag{2}$$

using the invariants $\phi_i = (1, \boldsymbol{\xi}, \xi^2/2)$, therefore obtaining

$$\mathcal{M}_1 f(\mathbf{x}, \boldsymbol{\xi}, t) = \iiint \phi_1(\boldsymbol{\xi}) f(\mathbf{x}, \boldsymbol{\xi}, t) \mathrm{d}^3\xi = \iiint 1 f(\mathbf{x}, \boldsymbol{\xi}, t) \mathrm{d}^3\xi = \frac{\rho}{m} \,, \tag{3}$$

$$\mathcal{M}_2 f(\mathbf{x}, \boldsymbol{\xi}, t) = \iiint \phi_2(\boldsymbol{\xi}) f(\mathbf{x}, \boldsymbol{\xi}, t) \mathrm{d}^3\xi = \iiint \boldsymbol{\xi} f(\mathbf{x}, \boldsymbol{\xi}, t) \mathrm{d}^3\xi = \frac{\rho}{m} \mathbf{u} \,, \tag{4}$$

$$\mathcal{M}_3 f(\mathbf{x}, \boldsymbol{\xi}, t) = \iiint \phi_3(\boldsymbol{\xi}) f(\mathbf{x}, \boldsymbol{\xi}, t) \mathrm{d}^3\xi = \iiint \frac{\xi^2}{2} f(\mathbf{x}, \boldsymbol{\xi}, t) \mathrm{d}^3\xi = \frac{\mathfrak{e}}{m} \,, \tag{5}$$

where ρ is the mass density, m is the particle mass, $\mathbf{u}$ is the velocity field and $\mathfrak{e}$ is the (total) energy density.

Solving the general form of the Boltzmann Equation, (1), is a rather complicated task which is usually done numerically. However, for the collisionless case, i.e. for $\mathcal{C} f(\mathbf{x}, \boldsymbol{\xi}, t)$, an analytical solution can be found, given by

$$f_{\mathrm{M}}(\mathbf{x}, \boldsymbol{\xi}, t) = \frac{\rho m^{\frac{1}{2}}}{[2\pi k_{\mathrm{B}} T]^{\frac{3}{2}}} \exp\left\{ -\frac{m}{2k_{\mathrm{B}} T(\mathbf{x}, t)} [\boldsymbol{\xi} - \mathbf{u}]^2 \right\} \,, \tag{6}$$

where k_{B} is the Boltzmann Constant and T is the temperature.

In this work we propose a new model to derive the MHD equations by introducing a complex-valued distribution function [3] which in the equilibrium case is given by

$$f_{\mathrm{M}}(\mathbf{x}, \boldsymbol{\xi}, t) = \frac{\rho m^{\frac{1}{2}}}{[2\pi k_{\mathrm{B}} T]^{\frac{3}{2}}} \exp\left\{ -\frac{m}{2k_{\mathrm{B}} T(\mathbf{x}, t)} \Big[\boldsymbol{\xi} - \Big(\mathbf{u} + i\mathbf{v}_{\mathrm{A}} \Big) \Big]^2 \right\} \,, \tag{7}$$

where $\mathbf{v}_{\mathrm{A}}$ is the Alfven Velocity given by

$$\mathbf{v}_{\mathrm{A}} = \frac{\mathbf{B}}{\sqrt{\mu\rho}} \,, \tag{8}$$

μ being the (local) magnetic permeability. As one can see, (7) maintains the general from of the real-valued equilibrium distribution function (6), while at the same time introducing the electromagnetic contributions.

Now it can be shown that, in a similar way as for (3)-(5), the magnetic field $\mathbf{B}$ may be also derived as a moment of the distribution function, such that (4) changes to

$$\Re \mathcal{M}_2 f(\mathbf{x}, \boldsymbol{\xi}, t) = \Re \iiint \phi_2(\boldsymbol{\xi}) f(\mathbf{x}, \boldsymbol{\xi}, t) \mathrm{d}^3\xi = \Re \iiint \boldsymbol{\xi} f(\mathbf{x}, \boldsymbol{\xi}, t) \mathrm{d}^3\xi = \frac{\rho}{m} \mathbf{u} \,, \tag{9}$$

$$\Im \mathcal{M}_2 f(\mathbf{x}, \boldsymbol{\xi}, t) = \Im \iiint \phi_2(\boldsymbol{\xi}) f(\mathbf{x}, \boldsymbol{\xi}, t) \mathrm{d}^3\xi = \Im \iiint \boldsymbol{\xi} f(\mathbf{x}, \boldsymbol{\xi}, t) \mathrm{d}^3\xi = \frac{\rho}{m} \mathbf{v}_{\mathrm{A}} \,, \tag{10}$$

Finally, taking the moments $\mathcal{M}_1$, $\mathcal{M}_2$, and $\mathcal{M}_3$ of the Boltzmann Equation using the complex-valued distribution function results in the following system of equations [3]:

$$\frac{\partial\rho}{\partial t}+\frac{\tau}{2}\frac{\partial^2\rho}{\partial t^2}+\mathrm{div}\rho\left(\mathbf{u}-\mathbf{w}\right)=0\,, \tag{11}$$

$$\frac{\partial\rho\mathbf{u}}{\partial t}+\frac{\tau}{2}\frac{\partial^2\rho\mathbf{u}}{\partial t^2}+\mathrm{div}\left[\rho\left(\mathbf{u}-\mathbf{w}\right)\times\mathbf{u}+\frac{B_iB_k}{\mu}\right]+\nabla\left(p+\frac{B^2}{2\mu}\right)=\mathrm{div}P_{\mathrm{NS}}\,, \tag{12}$$

$$\frac{\partial E}{\partial t}+\frac{\tau}{2}\frac{\partial^2 E}{\partial t^2}+\mathrm{div}\left[\left(E+p+\frac{B^2}{2\mu}\right)\left(\mathbf{u}-\mathbf{w}\right)\right]=\mathrm{div}\mathbf{q}+\mathrm{div}\left(P_{\mathrm{NS}}\mathbf{u}\right)\,, \tag{13}$$

$$\frac{\partial\mathbf{B}}{\partial t}+\frac{\tau_{\mathrm{m}}}{2}\frac{\partial^2\mathbf{B}}{\partial t^2}=\mathrm{rot}\left[\left(\mathbf{u}-\mathbf{w}\right)\times\mathbf{B}+\nu_{\mathrm{m}}\mathrm{rot}\mathbf{B}\right] \tag{14}$$

$$\mathrm{div}\mathbf{B}=0\,, \tag{15}$$

where

$$w_k=\frac{1}{\rho}\frac{\partial}{\partial x_i}\left[\left(p+\frac{B^2}{2\mu}\right)\delta_{ik}+\rho u_iu_k-\frac{B_iB_k}{\mu}\right]\,, \tag{16}$$

P_{NS} is the viscous stress tensor, $\mathbf{q}$ is the heat flux vector, ν_{m} is the magnetic viscosity and τ_{m} is defined via the relation

$$\frac{\tau_{\mathrm{m}}}{2\rho}\left(p+\frac{B^2}{2\mu}\right)=\nu_{\mathrm{m}}\,. \tag{17}$$

With that at hand, we were then able to apply the method to one of the most important problems in present day astrophysics and cosmology, namely to the question of the origin and time evolution of Intergalactic Magnetic Fields. As for their origin, there are mainly two scenarios discussed in the literature – on the one hand the cosmological one, where the magnetic field is produced by some process in the very early Universe, and on the other hand the cosmological one, where a seed of the magnetic field is created during structure formation and then amplified by some dynamo effect.

Here, we show results of the aforementioned application of our method – on the one hand, concerning the astrophysical scenario, the simulation of galactic winds, i.e. the ejection of matter from galaxies which might also carry magnetic energy. This occurs due to Supernova explosions which cause turbulence of the galactic matter and injects energy into it, such that it can leave the galaxy and transport momentum, energy and magnetic fields into the intergalactic medium.

On the other hand, for the cosmological scenario, we consider the time evolution of primordial magnetic fields and their possible imprints on the Cosmic Microwave Background (CMB). This happens due to the fact that stochastic magnetic fields with a given spectrum, created for example during quantumelectrodynamic or quantumchromodynamic phase transition. These magnetic fields then cause pressure gradients which create over- and underdensities of the underlying primordial matter. These density fluctuations influence the cosmic Recombination, which, in turn, creates anisotropies in the CMB which can be measured by modern instruments such as WMAP or Planck. Through a thorough statistical analysis a robust upper limit on the magnetic field strength may be placed.

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