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INCLUDING MODEL UNCERTAINTIES IN SEISMIC FULL WAVEFORM INVERSION

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Abstract. Full waveform inversion aims to estimate detailed maps of subsurface medium parameters from seismic data. It is typically cast as a non-linear least-squares problem involving a non-linear forward operator that simulates wave propagation in the earth. In this paper, we incorporate model uncertainties in the problem. This leads to an alternative formulation of full-waveform inversion that has several advantages. In particuar, the resulting method is more robust to errors in the forward operator and initialisation of the non-linear inversion procedure. A disadvantage is its high computational cost resulting from the need to invert a large dense matrix. We adress this issue by assuming a particular form of the covariance matrix corresponding to the modelling errors. This allows to use the Woodbury matrix identity to invert the dense matrix explicitly. Numerical examples illustrate the performance of the method.

1 INTRODUCTION

Full waveform inversion aims to estimate detailed maps of subsurface medium parameters m from seismic data d [3]. It is typically cast as a non-linear least-squares problem involving a non-linear forward operator F that simulates wave propagation in the earth

$$\min_m \sum_{i=1}^{n_s} \|F_i(m) - d_i\|^2, \tag{1}$$

with $\|\cdot\|$ denoting the L_2-norm. The seismic data consists of n_s shot-records, each consisting of n_r time-series. The forward operator simulates the data by solving a wave-equation with spatially varying coefficients, m, for a source q_i and sampling the resulting wavefield at all n_r receiver locations as a function of time. We can then formally define the forward operator as

$$F_i(m) = PA(m)^{-1}q_i. \tag{2}$$

For typical scenarios we have $n_s = n_r \sim 10^3 - 10^6$ and $n_t \sim 10^3$.

2 METHOD

Taking into account errors in the measurement process as well as the physics, we obtain the following extended formulation

$$\min_m \sum_{i=1}^{n_s} \|PA(m)^{-1}q_i - d_i\|^2_{\Sigma(m)}, \tag{3}$$

where $\|\cdot\|_W$ denotes a weighted L_2-norm. The weight is formally given by

$$\Sigma(m) = \left(PA(m)^{-1}\Sigma_p^{-1}A(m)^{-*}P^* + \Sigma_m\right)^{-1}, \tag{4}$$

where Σ_p and Σ_m represent the covariances corresponding to errors in the physics and the measurements. More details on the derivation can be found in [1].

In practical settings all quantities are discretised and we represent m on a 2D or 3D grid with n_x gridpoints and discretise the temporal dimension using n_t gridpoints. A shot-record can then be represented as an $M-$dimensional vector, with $M = n_t \times n_r$. This results in $\Sigma(m) \in \mathbb{R}^{M\times M}$. It is not feasible to explicitly compute this matrix as it would involve $N = n_t \times n_x$ forward simulations. Moreover, it would not be feasible to invert a dense matrix of this size. By choosing Σ_p^{-1} seperately for each source as $\Sigma_p^{-1} = q_i q_i^*$ we obtain a much more convenient weight-matrix

$$\Sigma_i(m) = \left(p_i p_i^* + \Sigma_m\right)^{-1}, \tag{5}$$

where $p_i = PA(m)^{-1}q_i$ represents the predicted data for the i^{th} source (which needs to be computed anyway). The inverse of this matrix can be computed explicitly using the Sherman-Morrison inverse formula:

$$\Sigma_i(m) = \Sigma_m^{-1} - \frac{\Sigma_m^{-1}p_i p_i^* \Sigma_m^{-1}}{1 + p_i^*\Sigma_m^{-1}p_i}, \tag{6}$$

which simplifies even further with the typical choice $\Sigma_m = \sigma_m I$.

The gradient of the objective in (3) can be computed using the adjoint-state method. Applying a steepest-descent method to minimize the objective, we end up with a basic method to solve (3). The main steps involved are *i) forward simulation* to compute $p_i = PA(m)^{-1}q_i$; *ii)* deconvolution (using (6)) to compute the weighted residual $\widetilde{r}_i = \Sigma_i(m)^{-1}(p_i - d_i)$; *iii) adjoint simulation* to compute the gradient g; *iv)* a *model update* to compute the next iterate $m \equiv m - \alpha g$ with stepsize $\alpha > 0$.

More details on the implementation can be found in [2].

3 RESULTS

Preliminary results indicate that incorporating uncertainties in the forward model can increase robustness of the approach to initialization of the non-linear inverse procedure [2]. Moreover, the computational cost of the proposed method is comparable to that of a conventional full-waveform inversion. The settings and results of the numerical experiments are presented in figures 1 and 2.

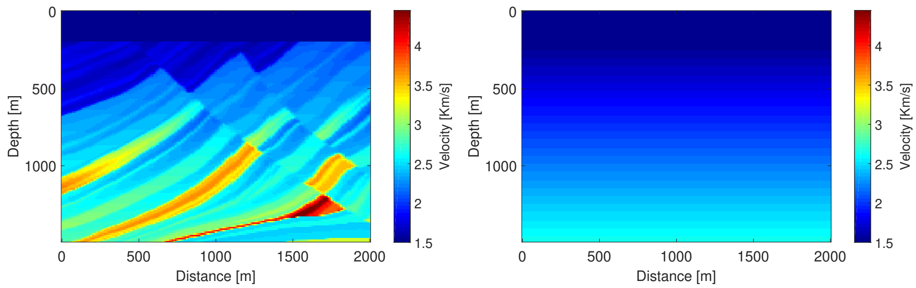

Figure 1: True (left) and initial (right) model used in the numerical example. Observed data are generated by simulating the response of $n_s = 10$ equispaced sources located at the surface using a finite-difference discretization of the wave equation in the temporal frequency domain (3 - 12 Hz). This effectively results in $n_t = 9$. The data are recorded at $n_r = 10$ equispaced receivers. For the inverse we use a multi-scale strategy, which proceeds from low to high frequencies. The stepsizes are computed using the Borzilai-Borwein rule.

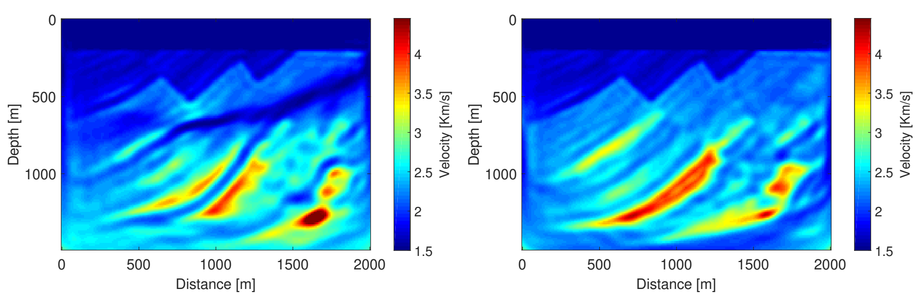

Figure 2: Result using conventional FWI (left) and the proposed method (right). We observe that the left result contains significant artifacts resulting from bad initialization. The result on the right is much closer to the true model and is about as good as one can expect using the current acquisition parameters.

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