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MARCHENKO MULTIPLE ELIMINATION ALGORITHM

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Abstract. The Marchenko multiple elimination scheme retrieves primary reflections in the two-way traveltime domain without model information or using adaptive subtraction. The set of coupled equations that define the problem are solved by an iterative algorithm. At each iteration, a convolution and correlation between the projected focusing function and the measured reflection response are performed. After each convolution or correlation, a truncation in the time domain is applied. After convergence, the resulting function is used to retrieve the physical primary reflections. We demonstrate that internal multiples are removed by using time-windowed input data that only contain primary reflections.

1 INTRODUCTION

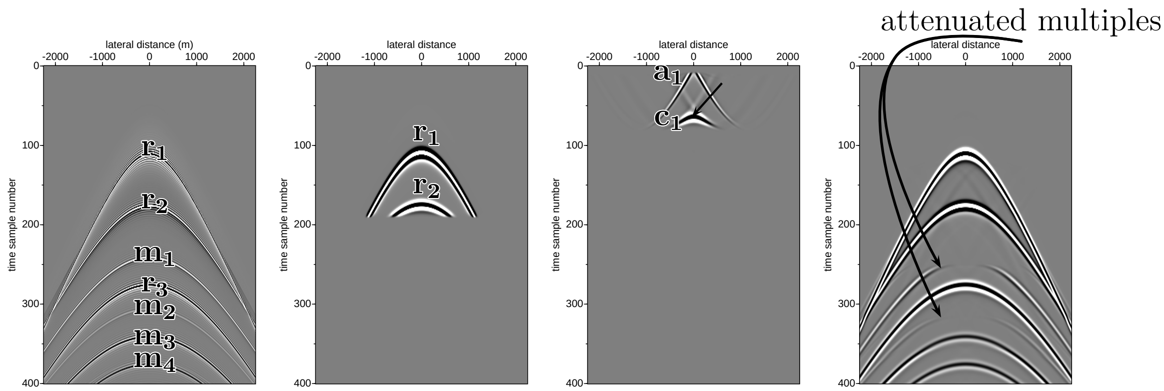
The Marchenko algorithm can eliminate internal multiple reflections in reflection data [1, 2]. The resulting fields can be used in imaging to create an image without reflection artifacts caused by internal multiples [3]. The theory of the Marchenko Multiple Elimination (MME) is based on the work of [7] and introduced in [4]. Following their notation the MME scheme is presented by

$$R_t(\mathbf{x}_0', \mathbf{x}_0'', t = t_2) = R(\mathbf{x}_0', \mathbf{x}_0'', t = t_2) + \sum_{m=1}^{\infty} M_{2m}(\mathbf{x}_0', \mathbf{x}_0'', t = t_2, t_2), \tag{1}$$

where R_t denotes the retrieved dataset without internal multiple reflections at constant time t_2 [5]. The reflection response $R(\mathbf{x}_0', \mathbf{x}_0'', t)$ at time t is measured with source and receiver positioned at $\mathbf{x}_0''$ and $\mathbf{x}_0'$, and is free from free-surface related multiple reflections and source wavelet. To explain the multiple elimination terms M_{2m} the summation in the right-hand side of equation 1 is divided into two parts and evaluated for all times t:

$$k_{1,i}^-(\mathbf{x}_0', \mathbf{x}_0'', t, t_2) = R(\mathbf{x}_0', \mathbf{x}_0'', t, t_2) - \sum_{m=1}^{i} \int_{t'=0}^{+\infty} \int_{\partial\mathbb{D}_0} R(\mathbf{x}_0''', \mathbf{x}_0', t') H(t - t' - \varepsilon) \times M_{2m-1}(\mathbf{x}_0''', \mathbf{x}_0'', t - t', t_2) \mathrm{d}\mathbf{x}_0''' \mathrm{d}t'. \tag{2}$$

$$M_{2m-1}(\mathbf{x}_0''', \mathbf{x}_0'', t - t', t_2) = \int_{t''=0}^{+\infty} \int_{\partial\mathbb{D}_0} R(\mathbf{x}_0, \mathbf{x}_0''', t'') H(t' - t + t_2 - t'' - \varepsilon) \times M_{2(m-1)}(\mathbf{x}_0, \mathbf{x}_0'', t - t' + t'', t_2) \mathrm{d}\mathbf{x}_0 \mathrm{d}t''. \tag{3}$$



a) $R(t)$ b) $M_0(-t)$ c) $M_1(t)$ d) $k^-_{1,30}(t)$

Figure 1: Creation of the event (labeled $\mathbf{c}_1$) that annihilates all the internal multiples between the first and second reflector. Picture a) shows the shot record with internal multiples $\mathbf{m}_i$, b) is M_0 created from a) by convolution with a wavelet and time truncation below sample 200. Picture c) shows M_1. Picture d) shows the partly annihilated multiples in $k^-_{1,30}$ after 30 iterations. Note that $k^-_{1,30}$ is a partial solution obtained for sample 200 and is not the final solution.

The constant ε indicates a small positive value which can be taken as the half source time-duration in practice. H indicates the Heaviside function, which is used to apply constant-time truncation window $(\varepsilon, t_2 - \varepsilon)$ in the equations. The acquisition surface is located at the surface boundary $\partial\mathbb{D}_0$. The minus superscript in equation 2 refers to upgoing wavefields at the surface. Equation 2 is the same as equation 1 with $k_1^-(t = t_2, t_2) = R_t(t = t_2)$ [6]. We can evaluate equation 2 also for $t \geq t_2 - \varepsilon$ and the equation can be further split in the time domain as follows

$$k^-_{1,i}(\mathbf{x}'_0, \mathbf{x}''_0, t, t_2) = \begin{cases} v^-_{1,i}(\mathbf{x}'_0, \mathbf{x}''_0, t, t_2) & t < t_2 - \varepsilon \\ u^-_{1,i}(\mathbf{x}'_0, \mathbf{x}''_0, t, t_2) & t \geq t_2 - \varepsilon \end{cases}. \tag{4}$$

The integral in equation 2 is a time domain *convolution* of R with M integrated over the spatial coordinate $\mathbf{x}'''_0$, which is the receiver position of the shot at $\mathbf{x}'_0$. Equation 3 is a time domain *correlation* of R with M integrated over the spatial coordinate $\mathbf{x}_0$, which is the receiver position of the shot at $\mathbf{x}'''_0$. The Heaviside functions H in these equations separate the convolution/correlation result into an update part ($v^-_{1,i}$ in equation 4) for the next M_{2m} and a solution part ($u^-_{1,i}$ in equation 4) to extract the time at t_2. In each iteration an updated field is computed by the integration of M_i with R.

The initialisation (M_0) of the scheme is a copy of the time reversed shot record, from which we would like to attenuate the internal multiples, and set to zero from the first sample 0 to sample $n_t - n_{t_2} + n_\varepsilon$, where n_t is the total number of samples. After the initialisation the algorithm switches between the computation of the integrals in equations 2 and 3. M_0 is inserted into equation 3, where a correlation and integration with R is computed and the time-windowed result gives M_1. This M_1 is inserted into equation 2, where it is convolved with R and the time-windowed result gives M_2 and the first coupled odd-even iteration is finished The algorithm continues by inserting M_2 into equation 3 and stops when convergence is reached, typically after 10-15 iterations.

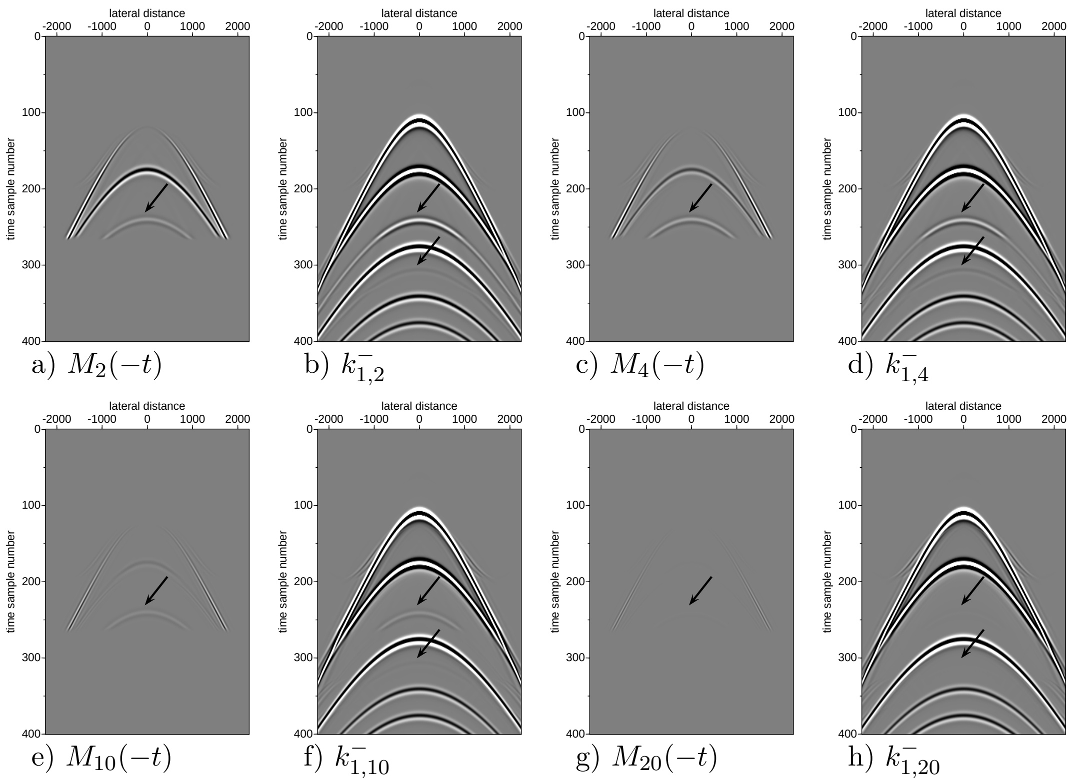


Figure 2: Updates for $k^-_{1,i}$ for a focal time at sample $n_{t_2} = 276$ after i iterations. The arrow indicates the first and second-order internal multiple between the first and second reflector.

2 MULTIPLE REMOVAL IN ACTION

The operation of the Marchenko algorithm is demonstrated with a four layer 1.5-dimensional horizontally layered model. The results in Figure 1 are partial solutions of the Marchenko equations computed for time sample $n_{t_2} = 200$. After applying the time window to the shot record in Figure 1a, which sets all samples in M_0 to zero beyond $200 - n_\varepsilon$, there are no internal multiple reflections present in M_0. The times between 0 and sample 200 include the primary reflections $\mathbf{r}_1$ and $\mathbf{r}_2$, but not $\mathbf{m}_1$, see Figure 1b. In the computation of M_1 one extra event ($\mathbf{c}_1$ pointed with an arrow in Figure 1c) is created to correct for the amplitude of the second reflector in k^-_1. This event is constructed from the correlation between R and M_0; $\mathbf{c_1} = \mathbf{r_2^*}.\mathbf{r_1} + \mathbf{m_1^*}.\mathbf{r_2}$, where the * means time-reversal. The amplitude of this event $\mathbf{c}_1$ converges to the amplitude that can annihilate the amplitude of the first multiple. Applying the converged $\mathbf{c}_1$ event on the reflection data through equation 3, causes that all multiples arising from bounces between the first and second reflector will vanish from the data in equation 1. The scheme finishes without ever having 'seen' the multiple; from $\mathbf{r}_1$ and $\mathbf{r}_2$ alone it created an event that can attenuate all the internal multiples between these reflectors. The arrows in Figure 1d, that shows $k^-_{1,30}(t)$, point at the multiples that are already partly gone. The multiples are only partly removed because only a small offset-range of $\mathbf{r}_2$ is used at sample 200. Repeating the scheme for samples larger than 200 will include larger offsets of $\mathbf{r}_1$ and $\mathbf{r}_2$ and attenuate also the higher offsets for all internal multiples between $\mathbf{r}_1$ and $\mathbf{r}_2$.

To demonstrate the effect of higher iterations the truncation time is chosen at sample 276: a first-order multiple of the second layer is now also present in the initialisation field M_0 after time-truncation. In the odd iterations the function $k^-_{1,i}(t)$, see equation 2, is updated with the odd $M_i(-t)$ terms and four selected iterations are shown in Figure 2. After two iterations all orders of multiples are predicted, but with incorrect amplitudes. In the following iterations the removal of higher-order multiples is improved because the removal of the first-order multiple improves. After 20 iterations the internal multiple events (indicated with arrows) have further attenuated and are not visible anymore; compare Figure 2b with 2h. The higher-order multiples do not have to be removed by extra events, but are removed automatically by removing the first-order multiple.

3 CONCLUSIONS

The MME algorithm has a straightforward mathematical expression, a simple numerical implementation, and can eliminate internal multiples from reflection data by only using time truncated shot records selected from that reflection data. Multiple annihilator events are computed from primary reflections only. These events eliminate the first-order internal multiples and hence the whole train of multiples associated with each first-order multiple is eliminated as well.

REFERENCES

[1] Slob, E., K. Wapenaar, F. Broggini and R. Snieder, Seismic reflector imaging using internal multiples with Marchenko-type equations. *Geophysics*, (2014) **79**(2):S63–S76.

[2] Behura, J., K. Wapenaar and R. Snieder, Autofocus imaging: Image reconstruction based on inverse scattering theory. *Geophysics*, (2014) **79**(3):A19–A26.

[3] Wapenaar, K., J. Thorbecke, J. van der Neut, F. Broggini, E. Slob and R. Snieder, Marchenko imaging. *Geophysics*, (2014) **79**(3):WA39–WA57.

[4] Zhang, L., J. Thorbecke, K. Wapenaar, and E. Slob, Transmission compensated primary reflection retrieval in data domain and consequences for imaging. *Geophysics*, (2019) **84**(4):Q27–Q36.

[5] Zhang, L., and M. Staring, Marchenko scheme based internal multiple reflection elimination in acoustic wavefield. *Journal of Applied Geophysics*, (2018) **159**(9):429–433.

[6] Thorbecke, J., L. Zhang, K. Wapenaar, and E. Slob, Implementation of the Marchenko Multiple Elimination algorithm. *Geophysics*, (2021) **86**(2):1–15.

[7] Van der Neut, J., and K. Wapenaar, Adaptive overburden elimination with the multidimensional Marchenko equation. *Geophysics*, (2016) **81**(5):T265–T284.