OBJECT-ORIENTED OPTIMIZATION FOR LARGE-SCALE SEISMIC INVERSION OF OCEAN-BOTTOM-NODE PRESSURE DATA

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Abstract. We present an object-oriented optimization framework for solving large-scale optimization problems of both convex and non-linear objective functions. The proposed framework allows for fast prototyping and testing on small-scale problems and seamlessly scales from local computers to high-performance-computing (HPC) facilities. We demonstrate its efficacy by using the proposed framework to solve a 3D acoustic full-waveform inversion problem of active seismic pressure data recorded by ocean-bottom nodes (OBNs) within the Gulf of Mexico.

1 INTRODUCTION

The concept of object-oriented non-linear optimization has been explored and implemented within multiple libraries [4, 6]. Other packages have explored the possibility of defining mathematical operations using the concept of operators for solving linear problems [5, 7]. We combine these two concepts and implement a Python optimization library that allows fast prototyping and testing to solve large-scale inverse problems. The proposed framework allows to easily combine operators whose computations are performed within general-purpose graphic processing units (GPGPUs) using the same code base. We show the effectiveness of the implemented library by solving a large-scale field-data full-waveform inversion (FWI) problem.

2 OBJECT-ORIENTED OPTIMIZATION

In the proposed inversion framework, we define different classes closely related to the mathematical entities commonly present in any inverse problem. The main two abstract objects composing our library are the vector and operator classes. The former represents a vector space element for which typical mathematical operations are defined (e.g., sum, scalar multiplication, norm). Any vector object can be transformed by applying a linear or non-linear operator, and it is implemented as an abstract object. This structure allows for the definition of useful methods for testing and combining operators that ultimately compose an inverse problem, which is represented by another abstract class. Finally, to solve an inverse problem, we define a solver object that takes a problem object and find its solution using any available iterative method. A list of examples of derived classes from the abstract objects is reported in Table 1.

Table 1: Examples of derived classes from the abstract parents.

Abstract class	Vector	Operator	Problem	Solver
Derived class	In-core	Linear	L2 linear	Linear CG
	Out-of-core	Non-linear	L2 regularized	Non-linear CG
	CuPy	Dask	L2 non-linear	L-BFGS
	Dask		L1 regularized	Split-Bregman

The implemented framework enables the definition of small- and large-scale inverse problems in which computations can be performed on a local machine or a large computer cluster, or cloud-based virtual machines using the same code base. Our proposed framework allows for fast prototyping and testing on simple examples and seamlessly portability to complex and computationally intensive inverse problems. The scalability of our objects relies on the Python library called Dask [2]. Compared to the Message Passing Interface (MPI), Dask presents better architecture deployment flexibility and is more resilient to system or hardware failures.

3 FULL-WAVEFORM INVERSION

FWI represents one of the most computational intensive inverse problems within the field of exploration geophysics [9]. FWI aims at recovering a model of the subsurface **m** by minimizing the following cost function,

$$\phi(\mathbf{m}) = \frac{1}{2} \|\mathbf{f}(\mathbf{m}) - \mathbf{d}\|_2^2, \tag{1}$$

where the non-linear operator \mathbf{f} is given by $\mathbf{f}(\mathbf{m}) = \mathbf{K}\mathbf{p}(\mathbf{m})$, in which \mathbf{K} represents the recording of the seismograms at the surface receivers, and \mathbf{d} is the observed data vector. The term \mathbf{p} represents the solution to a considered wave equation (e.g., elastic or acoustic isotropic wave equations). Thus, it is necessary to find the solution to a partialdifferential-equation-constrained optimization problem within any FWI methodology. In the reported field-data example, we retrieve an estimate of the 3D P-wave velocity model by solving an acoustic FWI problem. The acoustic isotropic wave equation is solved using a second-order-time and eighth-order-space finite-difference approach using multiple GPGPU cards. This high-performance operator is easily combined with other operator objects to solve the FWI problem using a spline parameterization for the model and a trace-by-trace normalization operation to mitigate the modeling inaccuracy compared to the real physics of the observed data [8, 1].

4 OBN FIELD-DATA APPLICATION

We employ the described inversion framework to solve the acoustic FWI problem defined by the pressure component of an OBN survey of the Gulf of Mexico. The horizontal domain extent is approximately 100 km², and the maximum depth is 4 km. We invert data from 41000 sources and 255 receivers for a maximum frequency of 18 Hz using the BFGS algorithm for 215 iterations [3]. Figure 1 shows the comparison between the initial and the final inverted P-wave velocity models. The FWI approach introduces multiple geologically consistent features. Also, a velocity decrease in the salt top is visible. This anomaly could be associated with salt-encased sediment packages included during the diapir formation. The computational performance of the GPU-based operator, in addition to the fast prototyping feature of our library, enabled us to quickly test different optimization methodologies to find the one that reached the best data fitting goal.



Figure 1: Comparison between the initial (top panels) and the inverted (bottom panels) acoustic velocity models for (a-c) z = 1.2 km and (b-d) x = 215.5 km.

5 CONCLUSIONS

We present an object-oriented framework that can be applied to large-scale optimization problems. The same code base permits fast prototyping and seamless scalability from local machines to HPC clusters. We demonstrate the efficacy of the proposed framework by solving a 3D field-data acoustic FWI problem of OBN pressure measured in the Gulf of Mexico.

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