MULTIDIMENSIONAL DECONVOLUTION FOR BOUNDARY REFLECTION REMOVAL AND COMPLETE SCATTERING CHARACTERIZATION IN PHYSICAL ACOUSTICS EXPERIMENTS

Xun Li^{*}, Theodor Becker^{*}, Matteo Ravasi[†], Johan Robertsson^{*}, and Dirk-Jan van Manen^{*}

* Institute of Geophysics, ETH Zurich, Sonneggstrasse 5, 8092 Zurich, Switzerland e-mail: xun.li@erdw.ethz.ch

[†]Earth Science and Engineering, KAUST, Thuwal, 23955, Kingdom of Saudi Arabia

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Abstract. In acoustic wave propagation experiments, waves reflected from the boundaries of the experimental setup often contaminate recorded data. We propose using multidimensional deconvolution (MDD) to post-process laboratory data such that the boundary-related scattering component is completely removed from recorded data and only the Green's functions associated with a scattering object of interest are obtained. The obtained Green's functions between any pair of points on a closed recording surface completely characterize an unknown scattering object inside the recording surface.

1 INTRODUCTION

Acoustic wave propagation experiments carried out in air or water often suffer from unwanted reflections at the boundaries of the experimental setup [2]. The undesired boundary reflections can interfere with, or mask, the wavefields related to the interior scatterers to be studied for their scattering properties.

In this paper, we apply multidimensional deconvolution (MDD), also called Rayleigh-Betti deconvolution, widely used in exploration seismology [4], to post-process data recorded in the laboratory such that the boundary-related scattering is removed, and an arbitrary scattering medium inside a closed-surface receiver array can be completely characterized. One key feature of the MDD method (used in a laboratory experiment) is that the material properties of the scattering medium (e.g., a rock sample) inside the receiver array do not need to be known.

2 METHOD

We carry out acoustic wave propagation experiments in a 2D waveguide according to the schematic shown in Fig. 1(a), and the corresponding physical setup shown in Fig. 1(b). The air-filled waveguide consists of two parallel plates and a rigid circular outer boundary placed between the two plates. An array of 52 loudspeakers (i.e., sources) is installed



Figure 1: (a) Schematic of the acquisition geometry for MDD experiments. A 2D waveguide is enclosed by a rigid boundary (solid black circle) on which evenly-spaced sources (red stars) are placed. Two circular layers of densely-spaced receivers (blue dots) make up the recording surface S^{rec} . (b) 2D waveguide for laboratory experiments. The scatterer under study is a steel block placed inside S^{rec} .

on the circular boundary (i.e., the emitting surface S^{emt}) while the pressure-sensitive microphones (i.e., receivers) are mounted flush with the inside of the plates. Note that the recording surface S^{rec} comprises two circular arrays, each consisting of 114 microphones.

To illustrate the theory for multidimensional deconvolution (MDD) [4], we consider a system of integral equations

$$p(\mathbf{x}_{r},t) = -2 \oint_{S^{rec}} p^{\mathrm{in}}(\mathbf{x}_{rec},t) * \overline{G}_{d}(\mathbf{x}_{r},t | \mathbf{x}_{rec},0) \, \mathrm{d}S(\mathbf{x}_{rec}) \tag{1}$$

where $p(\mathbf{x}_r, t)$ and $p^{\text{in}}(\mathbf{x}_{rec}, t)$ are the total and in-going pressures recorded and separated (at the inner circle of receivers) on the recording surface S^{rec} , and \overline{G}_d is the dipolar Green's function for a normally-oriented body force source and a pressure wavefield. The Green's functions are associated with a desired, unbounded experiment where the scatterers outside S^{rec} including the rigid boundary, are replaced by a homogeneous medium (i.e., radiation boundary conditions). The two-layer recording surface S^{rec} allows for wavefield decomposition, and a method modified from Ref.[1] is used for separating pressure wavefields recorded along a general curved surface into in-going (and out-going) components.

An MDD experiment involves sequentially exciting the sources on S^{emt} (Fig. 1). The source signature corresponds to a Ricker wavelet with peak frequency $f_p = 2$ kHz. From the recorded data for all the sources, a system of equations is assembled from Eq. (1) and is solved for the desired Green's functions using an iterative damped least-squares method [3].

3 RESULTS

Figure 2 shows examples of the data acquired in the waveguide, the separated ingoing wavefield, and the MDD results (inverted Green's functions) which are convolved with the source signature for visualization purposes only. The obtained Green's functions contain undesired direct arrivals as well as pole artefacts (around time t = 0) caused by the integrable singularity that exists for co-located source and receiver positions of



Figure 2: (a) Physical experiment with a source (red star) located at the rigid boundary (solid black circle) and receivers on S^{rec} (blue circle) with azimuth ϕ . The black rectangle denotes the steel block. (b, c) The data recorded for the scenario (a), and their separated in-going components. (d) A desired waveguide experiment with a radiation boundary condition (dashed black line). The cyan star denotes a dipolar source. (e) Inverted Green's functions associated to the graph (d). (f) Scattering Green's functions.

the Green functions $[\mathbf{x}_r = \mathbf{x}_{rec}$ for $\overline{G}_d(\mathbf{x}_r, t | \mathbf{x}_{rec}, 0)$ in Eq. (1)]. Hence, we carry out a second experiment without a scatterer inside S^{rec} and subtract the corresponding MDD result from that obtained in the (first) experiment with the scatterer present. We then obtain the scattering Green's functions that are only related to the steel object, as shown in Fig. 2(f). Figure 3(a) shows a zoom-in of the MDD results, which is compared to a reference solution (with amplitude normalization) obtained from counterpart synthetic experiments which are carried out using COMSOL Multiphysics[®]. Comparing Figs. 2(b) and (e) shows that almost all the boundary reflections are removed, and Figure 3 shows that the desired Green's functions only related to the steel scatterer can be effectively recovered by the MDD method.

4 DISCUSSION

Whilst not directly shown here, another appealing characteristic of the MDD theory is that sources are not necessarily located at the rigid boundary, and can be placed *anywhere* between the rigid boundary and the recording surface S^{rec} . The physical sources can be also of any characteristics without calibration, and their distribution can be even sparse without satisfying the spatial Nyquist sampling criterion.



 (ϕ)

Figure 3: (a) Zoom-in of laboratory MDD results [Fig. 2(f)]. (b) The synthetic reference solution.

5 CONCLUSIONS

A multidimensional deconvolution (MDD) method was proposed and applied to a 2D acoustic wave propagation experiment such that the scattering imprint related to the experimental domain boundary is completely removed. The MDD approach, together with the closed-surface acquisition geometry, enables the full characterization of scatterers placed inside an experimental domain.

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